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Advances in Mathematical and Statistical Science

Editors

Dr. Med Ram Verma
Dr. Megha Bhamare
Dr. Sheetal Gomkar
Dr. H. S. Tomar

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Editors

Dr. Med Ram Verma

Principal Scientist

ICAR-Indian Veterinary Research Institute, Izatnagar,

Bareilly, Uttar Pradesh

Dr. Megha A. Bhamare (Patil)

Department of Mathematics,

Krantiveer Vasantrao Narayanrao Naik Arts, Commerce and Science College,

Nashik, M. S.

Dr. Sheetal Gomkar

Department of Mathematics,

Janata Mahavidyalya,

Chandrpur, M.S

Dr. H. S. Tomar

Department of Mathematics,

Chintamani College of Arts and Science,

Gondpipri, Dist. Chandrpur, M.S.

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PREFACE

*We are delighted to publish our book entitled "**Advances in Mathematical and Statistical Science**". This book is the compilation of esteemed articles of acknowledged experts in the fields of basic and applied mathematical science.*

This book is published in the hopes of sharing the excitement found in the study of mathematics and statistical science. Mathematical science can help us unlock the mysteries of our universe, but beyond that, conquering it can be personally satisfying. We developed this digital book with the goal of helping people achieve that feeling of accomplishment.

The articles in the book have been contributed by eminent scientists, academicians. Our special thanks and appreciation goes to experts and research workers whose contributions have enriched this book. We thank our publisher Bhumi Publishing, India for taking pains in bringing out the book.

Finally, we will always remain a debtor to all our well-wishers for their blessings, without which this book would not have come into existence.

\- ***Editors***

CONTENT

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FACTORIZATION OF COMPLETE GRAPH AND ITS APPLICATION

Chandramani I* and Sakthivadivu M
Department of Mathematics,
Avinashilingam Institute for Home Science and Higher Education for Women,
Coimbatore – 641 043
*Corresponding author E-mail: icmpadhu@gmail.com

Abstract:

Factorization of a graph G is a set of spanning subgraphs of G that are pairwise edge disjoint. A graph is called k-factorizable if it can be represented as a union of edge-disjoint k factors. In this chapter we obtain factorization of complete graph and its application.

AMS Subject Classification: 05C69
Keywords: factorization, factors

1. Introduction

Graphs are the mathematical structure which consists of vertex set V and edge set E. It is used to model pair-wise relation between objects from a certain collection. Vertices are represented as points in the plane edges are represented as the line segments connecting them. Graphs are ever-present miniature of both from nature and man-made structures.

When any two vertices are joined by more than one edge, the graph is called a multi-graph. A graph without loops and with at most one edge between any two vertices is called a simple graph. Unless stated otherwise, graph is assumed to refer a simple graph. When each vertex connected by an edge to every other vertex, the graph is called a complete graph.

If two graphs G_1 and G_2 have the same vertex set, then the union $G_1 \cup G_2$ has the same vertex set and the edge set $E(G_1 \cup G_2)$ is $E(G_1) \cup E(G_2)$. If $E(G_1) \cap E(G_2) = \emptyset$ then $E(G_1) \cup E(G_2)$ may be termed the edge-disjoint union of $E(G_1)$ and $E(G_2)$. If two graphs G_1 and G_2 have disjoint vertex sets then the union of G_1 and G_2 is $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$. Partition [7] of G into edge - disjoint sub-graphs $G_1, G_2 ... G_r$ such that $E(G) = E(G_1) \cup E(G_2) \cup ... \cup E(G_r)$ is called decomposition of G and we write $G = G_1 \oplus G_2 \oplus ... \oplus G_r$.

If every pair of vertices are joined by an edge, we say that the graph is complete and if, in addition, $|V(G)| = n$, we denote this graph by K_n.

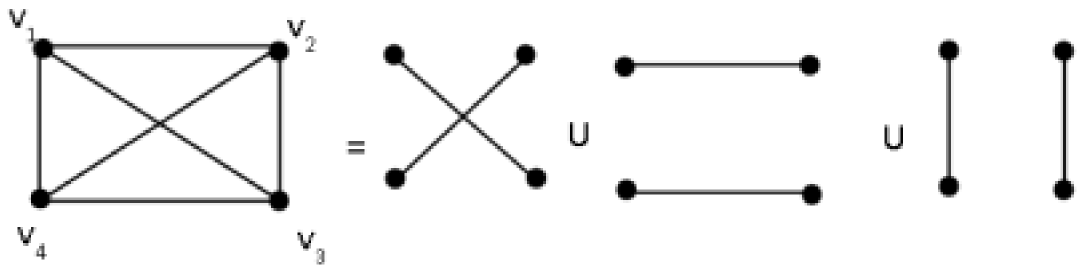
There is a vast body of work on factors and factorizations and this topic has much in common with other areas of study in graph theory. For example, factorization significantly overlaps the topic of edge coloring.Indeed, any color class of a proper edge coloring of a graph is just a matching. Moreover, the Hamilton cycle problem can be viewed as the search for a connected factor [1].

In the most general sense, a factor of a graph G is just a spanning sub-graph of G and a graph factorization of G is a partition of the edges of G into factors.

A factor F described in terms of its degrees will be called a degree factor. For example, if a factor F has all its degrees equal to 1, it is called a 1-factor (or a perfect matching). If the factor is described in some other graphical concept, it is called a component factor. If the edge set of a graph G can be represented as the edge-disjoint union of factors F_1, F_2, $F_3 \ldots F_K$. We refer to {F_1, F_2, $F_3 \ldots F_K$} as a factorization of graph G.

Factor of a graph G is a spanning sub-graph, k-factor of a graph is a spanning k-regular sub graph, and a k-factorization partitions the edges of the graph into disjoint k-factors. A graph G is said to be k-factorable if it admits a k-factorization.

1-factor is a sub-graph of a graph G where each of the vertices is of degree one and union of these sub-graphs forms the original graph. Suppose K_4 is a complete graph then the 1-factor as follows



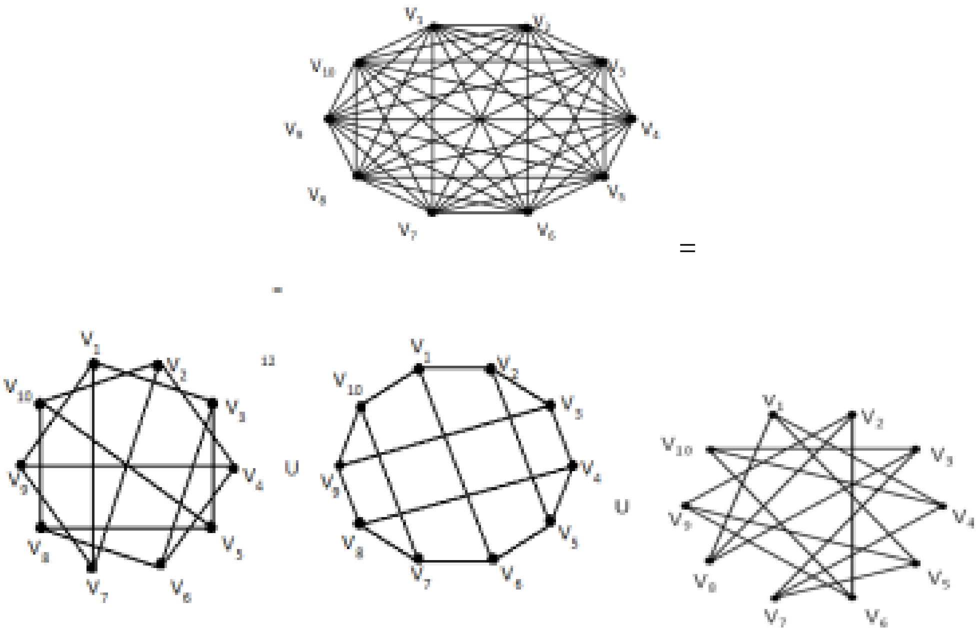
2. Factorization of Complete Graph

Theorem 2.1:

The Complete graph K_{6v-2}, $v \geq 1$has $2v-1$ number of 3-factor sub-graphs.

Proof:

Let us use induction method to prove this theorem. Consider the complete graph K_{6v-2} with v= 1. Then we have the complete graph K_4, which has 4 vertices and 6 edges. So, it has one 3-factor, which means it has one three regular sub-graph. Therefore, the theorem is true for v= 1. Next, we should prove that the theorem is true for v=2.If v=2, then we get a complete graph K_{10}, with 10 vertices and 45 edges. For this K_{10} the number of three factor sub-graphs is clearly three, and each three factor sub-graph has fifteen edges. As a result, the theorem holds for v=2.



If the process continues as same then the case is true for v=n. So we have the complete graph as $K_{6n-2,}$ in which the number of vertices is 6n−2 and total number of 3-factor sub-graphs are 2n−1. The theorem is true for v=n. Now our aim is to prove that the theorem is true for v=n+1.The following step is to prove that the theorem is true for v=n+1.Take v=n+1 then we get the graph K_{6n+4}. Let E(G) be the edge set of K_{6n+4}.we must give the partition of the edge set E(G) into the 3-factors. Let V(G) ={v_1, v_2, v_3…v_n, v_{6n+4}} be the vertex set of K_{6n+4}.

The set G={G_1, G_2…G_K} be the spanning sub-graph of $K_{6n+4.}$ A Graph K_{6n+4} is said to be factorable into G_1, G_2…G_K if each G_i where i =1,2,3…. k is a spanning sub-graph of K_{6n+4}.Then the set E(G) ={E(G_1), E(G_2), E(G_3), …E(G_i)} is pair-wise disjoint. Also, we get the following form

$$\bigcup_{i=1}^{k} E(G_i) = E(G)$$

For the complete graph K_4 (K_{6v-2}, where v= 1) there exist **one** (2v−1=2(1)−1=**1**) 3-factor sub-graph. For K_{10} (K_{6v-2}, where v=2) there exist **three** (2v−1=2(2)−1=**3**) 3-factor sub graphs. For K_{16} (K_{6v-2}, where v=3) there exist **five** (2v−1=2(3)−1=**5**) 3-factor sub-graph and so on. For K_{6n-2} (K_{6v-2}, where v=n) there exist **2n−1** (2v−1=**2n**−**1**) 3-factor sub graphs. So for K_{6n+4} (K_{6v-2}, where v=n+1) there exist **2n**+**1**(2v−1=2(n+1)−1=**2n**+**1**) 3-factor sub-graphs.

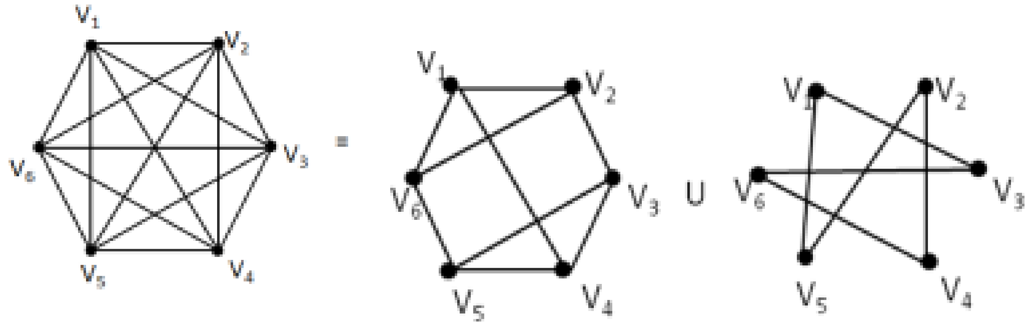
Therefore, a complete graph with 6n+4 vertices is factorized into 2n+1number of 3-factor sub-graphs and this 3-factorization partitions the edge set E(G) into disjoint 3-factors. As a result, the theorem is true for all v≥ 1.

Theorem 2.2:

The Complete graph K_{6v} where $v \geq 1$has $2v-1$ number of 3- factor sub-graph and one 2-factor sub-graph.

Proof:

With the use of the mathematical induction method, we will establish this theorem. Consider the Complete graph K_{6v} with v=1. Then there is the complete graph K_6, which has six vertices and fifteen edges. It has one 2-factor sub-graph and one 3-factor sub-graph, and when these two graphs are combined, we get K_6. As a result, the theorem holds for v=1.



Next, we should prove that the theorem is true for v=2.When we put v=2 we get the complete graph K_{12} which has 12 vertices and 66 edges. There are exactly three 3-factor sub graphs and one 2-factor sub-graph. There are 18 edges in each three-factor graph and 12 edges in each two-factor graph. As a result, the theorem holds for v=2.

If this process continues in this manner, then the theorem is true for v=n. So, we have complete graph K_{6n}, in which the number of vertices is 6n and the total number of 3-factor sub-graphs are $2n-1$ and there exist one 2-factor sub-graph. Then the theorem is true for v = n. Our aim is to prove that theorem is true for v = n+1.

If we substitute v = n+1, then we get K_{6n+6}. Let E(G) be the edge set of K_{6n+6}. We have to give the partition of the edge set E(G) into the 3-factors and 2-factor. Let V(G) ={ v_1, v2, $v_3, \ldots v_n, v_{6n+6}$ } be the vertex set of K_{6n+6} .The set G={G_1,G_2 ,…G_K} be the spanning sub graph of K_{6n+6} . A graph K_{6n+6} is said to be factorable into G_1, G_2 …G_K if each G_i where i=1,2,3…. k is a spanning sub-graph of K_{6n+6}. The set E(G) ={E(G_1), E(G_2), E(G_3), …E(G_i)} is pair-wise disjoint.

For the complete graph K_6 (K_{6v}, where v= 1) there exist **one** ($2v-1=2(1)-1=$**1**) 3-factor sub-graph and one 2-factor sub-graph. For K_{12} (K_{6v}, where v=2) there exist **three** ($2v-1=2(2)-1=$**3**) 3-factor sub-graphs and one 2-factor sub-graph. For K_{18} (K_{6v}, where v=3) there exist **five** ($2v-1=2(3)-1=$**5**) 3-factor sub-graphs and one 2-factor sub-graph and so on. For K_{6n}

(K_{6v}, where v=n) there exist **2n−1** (2v−1=**2n−1**) 3-factor sub-graphs and one 2-factor sub-graph. For K_{6n+6} (K_{6v} where v=n+1) there exist **2n+1** (2v−1=2(n+1)−1=**2n+1**) 3-factor sub-graphs and one 2-factor sub-graph.

Therefore, a complete graph with 6n+4 vertices is factorized into 2n+1 number of 3-factor sub-graphs and one 2-factor sub-graph. This 3-factorization and 2-factorization partitions the edge set E(G) into disjoint 3-factors and 2-factors

Hence the theorem is true for all v≥ 1.

Theorem 2.3:

The Complete graph K_{6v+2}, where v≥ 1has 2v number of 3- factor sub-graphs and one 1-factor sub-graph.

Proof:

With the use of the mathematical induction method, we will verify this theorem. Consider the complete graph K_{6v+2} with v=1 there is the complete graph K_8, which has eight vertices and twenty-eight edges. It has one 1-factor sub-graph and two 3-factor sub- graphs, and the union of these two graphs yields K_8. As a result, the theorem holds for v=1.

Next, we should prove that the theorem is true for v=2. Let v=2 then we get complete graph K_{14} with 14 vertices and 91 edges. It is obvious that the complete graph K_{14} has four 3-factor sub-graphs and one 1-factor graph. Each three-factor sub-graph has 21 edges, while each factor graph has seven. As a result, the theorem is true for v=2.Let us assume that the theorem is true for v=n. The complete graph thus takes the form K_{6n+2}, where the number of vertices is 6n+2 and the number of 3- factor sub-graphs is 2n and one 1-factor sub-graph. The theorem is true for v=n. Our aim is to prove that theorem is true for v=n+1.

Now, if we substitute v=n+1 then the complete graph becomes $K_{6(n+1)+2}$, with the number of vertices equaling 6(n+1)+2=6n+8. Let E(G) be the edge set of K_{6n+8}.We have to give the partition of the edge set E(G) into the 3-factors and 1-factor.V(G) ={v_1, v_2 , v_3,……v_n,v_{6n+8} } be the vertex set of K_{6n+8}.The set G={G_1,G_2 ,…G_K} be the spanning sub-graph of K_{6n+8} . A Graph K_{6n+8} is said to be factorable into G_1,G_2 ,…..G_K if each G_i where i=1,2,3….k is a spanning sub-graph of K_{6n+8}.The set E(G) ={E(G_1), E(G_2), E(G_3), …E(G_i)} is pair-wise disjoint.

For the complete graph K_8 (K_{6v+2}, where v= 1) there exist **two** (2v=2(1)=**2**) 3-factor sub graph and one 1-factor sub-graph. For K_{14} (K_{6v+2}, where v=2) there exist **four** (2v=2(2)=**4**) 3-factor sub-graphs and one 1-factor sub-graph. For K_{20} (K_{6v+2}, where v=3) there exist **five** (2v=2(3)=**6**) 3-factor sub-graphs and one 1-factor sub-graph and so on. For K_{6n+2} (K_{6v+2}, where v=n) there exist **2n** (2v=**2n**) 3-factor sub-graphs and one 1-factor sub-graph. For K_{6n+8} (K_{6v+2}, where v=n+1) there exist **2n+2** (2v=2(n+1)=**2n+2**) 3-factor sub-graphs and one 1-factor sub-graph.

Therefore, a complete graph with 6n+8 vertices is factorized into 2n+2 number of 3-factor sub-graphs and one 1-factor sub-graph. This 3-factorization and 2-factorization partitions the edge set E(G) into disjoint 3-factors and 1-factors.

Theorem 2.4:

For the Complete graph K_{4v+1}, v≥1, there exist v number of 4-factor sub-graph.

Proof:

We prove this theorem with the help of mathematical induction method. Here, the complete graph K_{4v+1} with v=1, we get K_5, which has five vertices and ten edges. Then the complete graph K_5 itself becomes one 4-factor sub-graph of K_5. Thus, a result, the theorem holds for v=1. Next, we should prove that the theorem is true for v=2.Take v=2, we get a complete graph K_9, with 9 vertices and 36 edges. The number of 4-factor sub-graphs is clearly two, and each 4-factor sub-graph has eighteen edges. As a result, the theorem is true for v=2.

If the process continues as same then the case is true for v=n. So we have complete graph as K_{4n+1}, in which the number of vertices is 4n+1 and the total number of 4-factor sub-graphs is v. The theorem is true for v=n. Our aim is to prove that the theorem is true for v=n+1.

Now, if we take v=n+1 then we get $K_{4(n+1)+1}$. Let E(G) be the edge set of K_{4n+5}.we have to give the partition of the edge set E(G) into the 4-factors. V(G) ={ v_1, v_2, v_3……v_n, v_{4n+5} } be the vertex set of K_{4n+5}. The set G={G_1, G_2 …G_K} be the spanning sub-graph of K_{4n+5}. A Graph K_{4n+5} is said to be 4-factorable into G_1, G_2 ...G_K if each G_i where i=1,2,3…. k is a spanning sub-graph of K_{4n+5}. then E(G) ={E(G_1), E(G_2), E(G_3), …E(G_i)} is pair-wise disjoint.

For K_5(K_{4v+1}, where v= 1) there exist **one** (v=**1**) 4-factor sub-graph. For K_9(K_{4v+1}, where v=**2**) there exist **two** (v=**2**) 4-factor sub-graphs.K_{13} (K_{4v+1}, where v=3) there exist **three** (v=**3**) 4-factor sub-graphs and so on. For K_{4n+1}(K_{4v+1}, where v=n) there exist **n** (v=**n)** 4- factor sub-graphs.K_{4n+5} (K_{4v+1}, where v=n+1) there exist **n+1** (v=**n+1**) 4-factor sub graphs.Therefore, a complete graph K_{4n+5} is factorized into n+1 number of 4-factor sub graphs and this 4-factorization partitions the edge set E(G) into disjoint 4-factors.

Theorem 2.5:

The Complete graph K_{4v+3} , v≥ 1has v number of 4-factor sub-graphs and one 2- factor sub graph.

Proof:

We prove this theorem with the help of mathematical induction method. Consider, the complete graph K_{4v+3}, v≥ 1and it has 4v+3 number of vertices.

Suppose v=1, then there is the complete graph K_7, which has seven vertices and twenty-one edges. Obviously, it has one 4-factor graph and one 2-factor sub-graph and union of these two graphs gives us K_7. As a result, the theorem holds for v=1.Next, we should prove that the theorem is true for v=2. If we put v=2, then we have the complete graph K_{11} in which the number of vertices equals to 11 and the number of edges equals to 55. Clearly it has two 4-factor sub-graphs and one 2-factor sub-graph. Hence the theorem is true for v=2.

If this process continues in this manner then the theorem is true for v=n. so we have complete graph K_{4n+3},in which the number of vertices is 4n+3 and the total number of 4- factor sub-graphs are n and there exist one 2-factor sub-graph. Then theorem is true for v = n. Our aim is to prove that theorem is true for v=n+1.

Now, if we substitute v=n+1 then the complete graph becomes $K_{4(n+1)+3}$, with the number of vertices equals to 4(n+1)+3=4n+7. Let E(G) be the edge set of K_{4n+7}.we have to give the partition of the edge set E(G) into the 4-factors and 2-factor. V(G) ={ v_1, v_2, v_3……v_n, v_{4n+7} } be the vertex set of K_{4n+7}. The set G={G_1, G_2…G_K} be the spanning sub-graph of K_{4n+7}. A Graph K_{4n+7} is said to be factorable into G_1, G_2….G_K if each G_i where i=1,2,3….k is a spanning sub-graph of K_{6n+8} . The set E(G) ={E(G_1), E(G_2), E(G_3), …E(G_i)} is pair-wise disjoint.

For the complete graph K_7 (K_{4v+3}, where v= 1) there exist **one** (v=**1**) 4-factor sub-graph and one 2-factor sub-graph. For K_{11} (K_{4v+3}, where v=2) there exist **two** (v=**2**) 4-factor sub graphs and one 2-factor sub-graph. For K_{18} (K_{4v+3}, where v=3) there exist **three** (v=**3**) 4- factor sub-graphs and one 2-factor sub-graph and so on. For K_{4n+3} (K_{4v+3} , where v=n) there exist **n** (v=**n**) 3-factor sub-graphs and one 2-factor sub-graph. For K_{4n+7} (K_{4v+3} where v=n+1) there exist **n**+**1** (v=**n**+**1**) 3-factor sub-graphs and one 2-factor sub-graph.

Therefore, a complete graph K_{4n+7} is factorized into n+1 number of 4-factor sub-graphs and one 2-factor sub-graph. This 4-factorization and 2-factorization partitions the edge set E (G) into disjoint 4-factors and 2-factors

3. Application of complete graph factorization

Graph is an abstract idea of representing any objects which are connected to each other in a form of relation. Graph partition is a technique to distribute the whole graph data as a disjoint subset to a different device. The need of distributing huge graph data set is to process data efficiently and faster the process of any graph related applications. It always aims to reduce the communication between machines in their distributed environment and distribute vertices roughly equal to all the machines [2].

3.1. Hamiltonian circuit

A graph structures can be extended by assigning a weight to each edge of the graph. Graphs with weights or weighted graphs are used to represent structures in which pair wise connections have some numerical values. For example, if a graph represents a road network, the weight could represent the length of the road. A digraph with weighted edges in the context of graph theory is called a network. In case of modeling and analyzing the traffic signals networks.

In 2010, Dutta et al developed some theorems about the application of regular planar sub graphs of the complete graphs and he studied various types of Hamiltonian circuits and edge disjoint Hamiltonian circuits of different types of regular sub-graphs of complete graphs. A Hamiltonian circuit in a graph is a closed path that visits every vertex in the graph exactly once. (Such a closed loop must be a cycle). A Hamiltonian circuit ends up at the vertex from where it started [8, 9].

Hamiltonian graphs are generally found to be very important in graph theory in which one must study the Hamiltonian circuit with weights related to minimum distance, time, cost etc. from the weighted graphs. Finding the Hamiltonian circuit with least cost route optimization problem in graph theory in which the nodes (cities) of a graph are connected by edges (routes), where the weight of an edge indicates the distance between two cities. The problem is to find a path that visits each city once, returns to the starting city, and minimize the distance traveled [6,10].

Here we formulate algorithm for its application. That is to find the Hamiltonian circuit with least distance for the given complete graph in which nodes (schools) are connected by the edges (route) where the weight of an edge indicates the distance between two schools. When we do the factorization, the given complete graph is reduced into regular sub-graph. From the obtained factors we can find the Hamiltonian circuit with least distance.

3.2. Algorithm:

This case includes the complete graph of the form K_{4v+3}, K_{4v+1} where $v \geq 1$, having the odd number of vertices.

Input:

Let G be the complete graph having vertex 4v+3 or 4v+1, $v \geq 1$.

Output:

To find the Hamiltonian Circuit with least distance.

Step1:

Assign weights for all the non-repeated edges for the complete graph K_{4v+3}, where v≥ 1.

Step2:

If there exist at least one 4-factor sub-graph and one 2-factor sub-graph for graph having 4v+3 number of vertices, then select two edges which should be minimum weighted.

Step3:

Draw the 2-factor sub-graph with the minimum weighted edges obtained in step2.

Step4:

Find the Hamiltonian circuit with the least distance. Stop the procedure. Suppose that the complete graph is of the form K_{4v+1}, where v≥ 1then go to step 5.

Step 5:

Assign the weights for all the non-repeated edges for the complete graph K_{4v+1}, where v≥ 1.

Step 6:

If there exist at least one 4-factor sub-graph having 4v+1 vertex, then select four edges which should be minimum weighted among all the weighted edges.

Step7:

Draw the 4-factor sub-graph with the minimum weighted edges obtained in step number 6, and then find the Hamiltonian circuit with the least distance.

Example:

The squads are going for an inspectional visit at Government and Corporation Schools located in the Coimbatore district.

Here are the schools and the distance (in KM) between each school is tabulated. There are seven alphabets assigned to represent the name of the schools.

A-Corporation Girls Higher Secondary School, R.S. Puram

B- Corporation Girls Higher Secondary School, Ram Nagar

C-- Corporation Girls High School, Sundakamuthur road, Selvapuram.

D-C.C.H. S, Variety Hall Road, Town Hall

E- Coimbatore Corporation Girls Secondary School, Arokiyasamy road, R.S. Puram

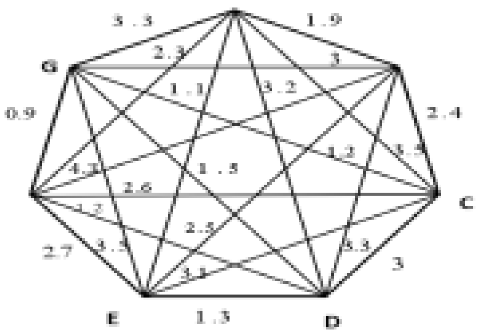
F-Government Girls Higher Secondary School, ThermuttiVeethi, Town Hall

G- Coimbatore Corporation Girls Secondary School, Oppanakarastreet, Town hall

SCHOOL	A	B	C	D	E	F	G
A	-	1.9	3.5	3.2	1.1	2.3	3.3
B	1.9	-	2.4	3.3	2.5	4.3	3
C	3.5	2.4	-	3	3.1	2.6	1.2
D	3.2	3.3	3	-	1.3	1.2	1.5
E	1.1	2.5	3.1	1.3	-	2.7	3.5
F	2.3	4.3	2.6	1.2	2.7	-	0.9
G	3.3	3	1.2	1.5	3.5	0.9	-

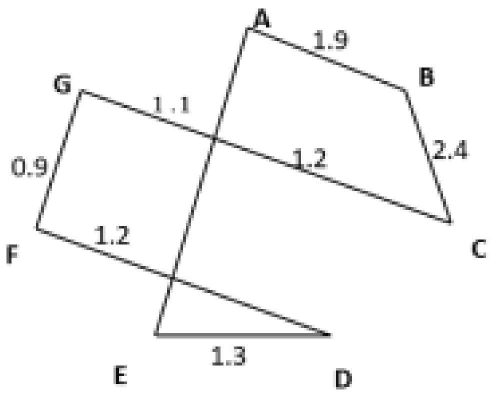
Table 3.1

From the Table-3.1, we have a complete graph of seven vertices, which is shown in Figure 3.3 and we apply the statement of the algorithm and find the least cost route we get complete graph for this table as follows



Now, applying the algorithm we obtain the 2-factor sub-graph which is give below

We have the minimum weighted Hamiltonian circuit as follow:



A → E → D → F → G → C → B → A

The total weight calculated as 1.1+1.3+1.2+0.9+1.2+2.4+1.9=10 kilometers.

Conclusion:

We obtained factorization of complete graphs with odd and even number of vertices into 1 – factor, 2–factor, 3 –factor and 4–factor sub-graphs. Also we discussed about some application of factorization of complete graphs and by the results obtained, we focused to find a least cost Hamiltonian circuit. The research of factorization of complete graphs is purely mathematical perspective so that all the definitions and theorems described in this section are accessible to Applied Mathematicians and Engineers for developing its practical applications.

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INTUITIONISTIC FUZZY $\pi\beta$ GENERALIZED CLOSED SETS

Prema S* and Nandhitha K
Department of Mathematics,
Avinashilingam Institute for Home Science and Higher Education for Women,
Coimbatore, Tamilnadu, India
*Corresponding author E–mail: premaparameswaran14@gmail.com

Abstract:

This chapter is devoted to the study of intuitionistic fuzzy topological spaces. In this chapter $\pi\beta$generalized closed sets in intuitionistic fuzzy topological spaces is introduced. The main objective of this chapter is to find the relationship between basic intuitionistic fuzzy sets and intuitionistic fuzzy $\pi\beta$ generalized closed and open sets. Also, we have analyzed some properties of $\pi\beta$ generalized closed sets in intuitionistic fuzzy topological spaces.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy $\pi\beta$ generalized closed sets, intuitionistic closed sets.

1. Introduction:

In 1965, the concept of Fuzzy sets was introduced by Lofti A. Zadeh [10] and in 1968, Chang[2] introduced and developed fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors conducted researchers on the generalization of these notions. In the year 1986, the notion of intuitionistic fuzzy sets was introduced by Atanassov[1] as a generalization of fuzzy sets and Coker[3] introduced the concept of intuitionistic fuzzy topological spaces in 1997. In the year 2014, Jayanthi D [5] has introduced intuitionistic fuzzy generalized β closed sets and Saranya M and Jayanthi D[7], has introduced intutionistic fuzzy β generalized closed sets in 2016. In this chapter, we have introduced the concept of intuitionistic fuzzy πβ generalized closed sets and investigated some of their properties and obtained some interesting characterizations.

2. Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An *intuitionistic fuzzy set*(IFS in short) A in X is an object having the form A = $\{<x,\mu A(x),\nu A(x)>/x \in X\}$where the functions$\mu A(x)$:X →[0,1] and $\nu A(x)$:X→[0,1] denote the degree of membership (namely $\mu A(x)$) and the degree of non – membership (namely $\upsilon A(x)$) of each element x ϵ X to the set A, respectively, and $0 \leq \mu A(x)+\upsilon A(x)\leq 1$ for each x ϵ X. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form A= $\{<x,(x),\nu A(x)>/x \ \epsilon \ X\}$ and B= $\{<x,\mu B(x),\nu B(x)>/x \ \epsilon \ X\}$. Then

a) A ⊆ B if and only if $\mu A(x)\leq\mu B(x)$ and $\nu A(x)\geq\nu B(x)$ for all x ϵ X

b) A=B if and only if A ⊆ B and B ⊆ A

c) Ac = $\{<x,\nu A(x),\mu A(x)>/x \ \epsilon \ X\}$

d) A ∩B = $\{<x,\mu A(x)\wedge\mu B(x),\nu A(x)\vee \nu B(x)>/x \ \epsilon \ X\}$

e) A ∪ B = $\{<x,\mu A(x)\vee\mu B(x),\nu A(x)\wedge \nu B(x)>/x \ \epsilon \ X\}$

For the sake of simplicity, we shall use the notation A= $<x,(\mu A,\mu B),(\nu A,\nu B)>$ instead of A= $<x,(A/\mu A,B/\mu B),(A/\nu A,B/\nu B)>$.

The intuitionistic fuzzy sets 0∼ = $\{<x,0,1)>/x \ \epsilon \ X\}$ and 1∼ = $\{<x,0,1)>/x \ \epsilon \ X\}$ are respectively the empty set and the whole set of X.

Definition 2.3.: [2] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

i i. 0~, 1~ ϵ τ

ii ii. G1 ∩ G2 ϵ τ, for any G1, G2 ϵ τ

iii iii. ∪ Giϵ τ for any family {Gi / i ϵ J} ⊆ τ.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement Ac of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and A =$<x,,,>$be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

int(A) = ∪ { G / G is an IFOS in X and G ⊆ A },

cl(A) = ∩ { K / K is an IFCS in X and A ⊆ K }.

Note that for any IFS A in (X, τ), we have cl(Ac) = (int(A))c and int(Ac) = (cl(A))c.

Definition 2.5.: [6] An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy regular open set* (IFROS in short) if A = int(cl(A)),

(ii) *intuitionistic fuzzy regular closed set* (IFRCS in short) if A = cl(int(A)).

Definition 2.6: [6] An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy semi open set* (IFSOS in short) if A ⊆ cl(int(A),

(ii) *intuitionistic fuzzy semi closed set* (IFSCS in short) if int(cl(A)) ⊆ A.

Definition 2.7: [6] An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy α-open set* (IFαOS in short) if A ⊆ int(cl(int(A))),

(ii) *intuitionistic fuzzy α-closed set* (IFαCS in short) if cl(int(cl(A)) ⊆ A.

Definition 2.8: [6] An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy pre open set* (IFPOS in short) if A ⊆ int(cl(A)),

(ii) *intuitionistic fuzzy pre closed set* (IFPCS in short) if cl(int(A)) ⊆ A.

Definition 2.9: [9] The union of IFROSs is called *intuitionistic fuzzy π-open set* (IFπOS in short) of an IFTS (X, τ). The complement of IFπOS is called *intuitionistic fuzzy π-closed set* (IFπCS in short).

Definition 2.10: [5] An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy β -open set* (IFβOS in short) if A ⊆ cl(int(cl(A))) .

(ii) *intuitionistic fuzzy β-closed set* (IFβCS in short) if int(cl(int(A))) ⊆ A.

Definition 2.11: [5] Let A be an IFS in an IFTS in (X,τ). Then the intuitionistic fuzzy β- interior and intuitionistic fuzzy β-closure of A are defined by

i i. βint(A)= ∪ {G/G is an IFβOS in X and G⊆ A},

ii ii. βcl(A) = ∩{K/K is an IFβCS in X and A⊆K}.

Note that for any IFS A in (X,τ), we have βcl(Ac) = (βint(A))c and βint(Ac)= (βcl(A))c.

Definition 2.12: [5] Let A be an IFS in (X,τ), then

i i. βcl(A)⊇ A ∪ int(cl(int(A)))

ii ii. βint(A)⊆ A∩ cl(int(cl(A)))

Definition 2.13: **[**8]

An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized closed set* (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X. The complement of IFGCS is called ***intuitionistic fuzzy generalized open set*** (IFGOS in short).

An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized open set* (IFGOS in short) if Ac is an IFGCS in X.

Definition 2.**14: [**8] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized open set* (IFGOS in short) if Ac is an IFGCS in X.

Definition 2.15: [5] An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy generalized β closed sets(IFGβCS for short) if βcl(A)⊆U and U is an IFOS in (X,τ). The familyof all IFGβCSs of an IFTS (X,τ) is denoted by IFGβC(X).

Definition 2.16: [7] An IFS A is an IFTS (X,τ) is said to be an intuitionistic fuzzy β generalized closed set (IFβGCS for short) if βcl(A) ⊆ U whenever A ⊆ U and U is an IFβOS in (X,τ). The complement Ac of an IFβGCS A is an IFTS (X, τ) is called intuitionistic fuzzy β generalized open set (IFβGOS in short) in X.

Definition 2.17: [4] An IFS A in (X,τ) is called an *intuitionitic fuzzy nowhere dense set* if there exist no IFOS U such that U $\subseteq cl(A)$. That is int(cl(A))=0~

Definition 2.18: [8] Two IFSs are said to be q-coincident (AqB in short) if and only if there exists an element x ϵ X such that (x) > (x) or $\nu(x)$ < μB (x).

Definition 2.19: [5] For any two IFSs A and B are said to be not q- coincident ($A\bar{q}$B) if and only if A ⊆ Bc.

Definition 2.20: [4] An intuitonistic fuzzy point (IFP in short) written as $(\alpha,)$ is defined to be an IFS of X given by

(α,β)= {$(\alpha,)$ if x=p (0,1) $otherwsie$

An intuitionistic fuzzy point $(\alpha,)$ is said to belong to as a set A if $\alpha \leq \mu$A and $\beta \geq \vartheta$A.

Definition 2.21: [3] Let (X, τ) be an IFTS and A, B be IFSs in X. Then the following properties hold:

i i. int(A) ⊆ A

ii ii. A ⊆ cl(A)

iii iii. A⊆ B ⟹cl(A) ⊆ cl(B)

iv iv. A⊆ B ⟹ int(A) ⊆ cl(B)

v v. int(int(A)) = int(A)

vi vi. cl(cl(A)) = cl(A)

vii vii. int(A∩B) = int(A) ∩ int(B)

viii viii. cl(A∪B) = cl(A) ∪ cl(B)

ix ix. int(1∼) = 1∼

x x. cl(0∼) = 0∼

3. Intuitionistic fuzzy $\pi\beta$ generalized closed sets

In this section we have introduced intuitionistic fuzzy $\pi\beta$ generalized closed sets and studied some of its properties.

Definition 3.1:

An IFS A in (X,) is said to be an intuitionistic fuzzy πβ *generalized closed sets* (IFπβGCS in short) if βcl(A)⊆ U whenever A ⊆ U and U is an $IF\pi OS$ in (X, τ).

Example 3.2:

Let X ={a,b} and let τ ={0~,G1,G2,1~} is an IFT on X, where G1=<x,(0.4a,0.5b),(0.6a,0.5b)> and G2=<x,(0.5a,0.6b),(0.5a,0.4b)> . Then the IFS A =<x,(0.4a,0.5b),(0.6a,0.5b)> is an IFπβGCS in (X, τ).

Theorem 3.3:

Every intuitionistic fuzzy closed set (IFCS in short) in (X,) is an IF$\pi\beta$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFCS and let A ⊆ U and U be an IFπOS in (X,). As (A)⊆$cl(A)$ = A ⊆U. We have (A)⊆U. Therefore A is an IFπβGCS.

Example 3.4:

Let X ={a, b} and let τ ={0~,G1,G2,1~} is an IFT on X, where G1=<x,(0.4a,0.5b),(0.6a,0.5b)> and G2=<x,(0.5a,0.6b),(0.5a,0.4b)> . Then the IFS A =<x,(0.4a,0.5b),(0.6a,0.5b)> is an IFπβGCS in (X, τ) but not an IFCS in X as cl(A)=G1c ≠ A.

Theorem 3.5:

Every intuitionistic fuzzy regular closed set (IFRCS in short) in (X,) is an IF$\pi\beta$GCS in (X, τ) but not conversely.

Proof:

Since every IFRCS is an IFCS.Hence A is an IF$\pi\beta$GCS in (X,).

Example 3.6:

Let X ={a,b} and let τ ={0~,G1,G2,1~} is an IFT on X, where G1=<x,(0.4a,0.5b),(0.6a,0.5b)> and G2=<x,(0.5a,0.6b),(0.5a,0.4b)> . Then the IFS A =<x,(0.4a,0.5b),(0.6a,0.5b)> is an IF$\pi\beta$GCS in (X, τ) but not an IFRCS in X as cl(int(A))=cl(G1)= G1c ≠ A.

Theorem 3.7:

Every intuitionistic fuzzy semi closed set (IFSCS in short) in (X, τ) is an IF$\pi\beta$GCS in (X,) but not conversely.

Proof:

Let A be an IFSCS and let A ⊆ U and U be an IFπOS in (X,). As (A)⊆$Scl(A)$ = A ⊆U, by hypothesis . Hence (A)⊆U. Therefore A is an IF$\pi\beta$GCS in (X,).

Example 3.8:

Let X ={a,b} and let τ ={0~,$G1$,$G2$,1~} is an IFT on X, where $G1$=<x,(0.5a,0.6b),(0.5a,0.4b)> and $G2$=<x,(0.2a,0.2b),(0.8a,0.8b)> . Then the IFS A =<x,(0.4a,0.6b),(0.6a,0.4b)> is an IF$\pi\beta$GCS in (X, τ) but not an IFSCS in X as int(cl(A)=int(G2c)= G1$\nsubseteq$ A.

Theorem 3.9:

Every intuitionistic fuzzy α closed set (IFαCS in short) in (X,) is an IF$\pi\beta$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFαCS and let A ⊆ U and U be an IFπOS in (X,). As (A)⊆$\alpha cl(A)$ = A⊆U. By hypothesis (A)⊆U. Therefore A is an IFπβGCS in (X,).

Example 3.10:

Let X ={a,b} and let τ ={0~,$G1$,$G2$,1~} is an IFT on X, where $G1$=<x,(0.4a,0.5b),(0.6a,0.5b)> and $G2$=<x,(0.5a,0.6b),(0.5a,0.4b)> . Then the IFS A =<x,(0.4a,0.5b),(0.6a,0.5b)> is an IF$\pi\beta$GCS in (X, τ) but not an IFαCS in X as cl(int(cl(A)))=cl(int(G1c)=cl(G1)= G1c $\nsubseteq$A.

Theorem 3.11:

Every intuitionistic fuzzy pre closed set (IFPCS in short) in (X,) is an IF$\pi\beta$GCS in (X,τ) but not conversely.

Proof:

Let A be an IFPCS and let A ⊆ U and U be an IFπOS in (X,). As (A) ⊆ pcl(A)= A⊆U. By hypothesis (A) ⊆ U. Therefore A is an IF$\pi\beta$GCS in (X,).

Example 3.12:

Let X ={a,b} and τ ={0~,$G1$,$G2$,1~} is an IFT on X, where $G1$=<x,(0.4a,0.5b),(0.6a,0.5b)> and $G2$=<x,(0.5a,0.6b),(0.5a,0.4b)> . Then the IFS A =<x,(0.4a,0.5b),(0.6a,0.5b)> is an IF$\pi\beta$GCS in (X, τ) but not an IFPCS in (X, τ) as cl(int(A))=cl(G1)= G1c$\nsubseteq$ A .

Theorem 3.13:

Every intuitionistic fuzzy πclosed set (IFπCS in short) in (X,) is an IFπβGCS in (X,) but not conversely.

Proof:

Let A be an IFπCS in (X, τ) and let A ⊆ U.Since every IFπCS is an IFCS. Therefore A is an IFπβGCS (X, τ).

Example 3.14:

Let X ={a,b} and let τ ={$0\sim$,$G1$,$G2$,$1\sim$}is an IFT on X, where $G1$=<x,($0.4a$,$0.5b$),($0.6a$,$0.5b$)> and $G2$=<x,($0.5a$,$0.6b$),($0.5a$,$0.4b$)> . Then the IFS A =<x,($0.4a$,$0.5b$),($0.6a$,$0.5b$)> is an IF$\pi\beta$GCS in (X, τ) but not an IFπCS in (X τ) as cl(int(A)) = cl(G1) = G1c ≠ A.

Theorem 3.15:

Every Intuitionistic fuzzy β closed set (IFβCS *in short*) in (X, τ) is an IFπβGCS in (X, τ) but not conversely.

Proof:

Let A be an IFβCS in (X, τ) and let A ⊆U and U be an IFπOS in (X, τ) . As (A)=A⊆ U, by hypothesis. Therefore A is an $IF\pi\beta GCS$ in (X, τ)

Example 3.16:

Let X ={a,b} and let τ ={$0\sim$,$G1$,$G2$,$1\sim$} is an IFT on X, where G1=<x,(0.4a,0.3b),(0.6a,0.7b)> and $G2$=<x,($0.5a$,$0.4b$),(0.5,$0.6b$)> . Then the IFS A =<x,($0.4a$,$0.6b$),($0.6a$,$0.4b$)> is an IF$\pi\beta$GCS in (X, τ) but not an IFβCS in (X,τ) as int(cl(int(A)))= int(cl(G1)) = int(G2c)= G2 ⊈ A .

Theorem 3.17:

Every intuitionistic fuzzy generalized closed set (IFGCS in short) in (X, τ) is an IFπβGCS in (X,) but not conversely.

Proof:

Let A be an IFGCS and let A ⊆ U and U be an IFπOS in (X, τ). As (A)⊆cl(A) ⊆U. We have (A)⊆U. Therefore A is an IFπβGCS in (X,).

Example 3.18:

Let X ={a,b} and let τ ={0∼,G1,G2,$1\sim$} is an IFT on X, where G1=<x,(0.4a,0.5b),(0.6a,0.5b)> and G2=<x,(0.5a,0.6b),(0.5a,0.4b)> . Then the IFS A =<x,(0.4a,0.5b),(0.6a,0.5b)> is an IFπβGCS in (X, τ) but not an IFGCS in X as cl(A)=G1c ⊈ G1,G2 where A ⊆ G1, G2.

Theorem 3.19:

Every intuitionistic fuzzy generalized pre closed set (IFGPCS in short) in (X, τ) is an IF$\pi\beta$GCS in (X,) but not conversely.

Proof:

Let A be an IFGPCS and let A ⊆ U and U be an IFπOS in (X, τ). Now (A) ⊆ p(A) ⊆ U, by hypothesis, which implies β(cl(A) ⊆U. Therefore A is an IF$\pi\beta$GCS in (X, τ).

Example 3.20:

Let X ={a,b} and let τ $=\{0\sim,G1,G2,1\sim\}$is an IFT on X, where $G1=<x,(0.4a,0.5b),(0.6a,0.5b)>$ and $G2=<x,(0.5a,0.6b),(0.5a,0.4b)>$. Then the IFS A $=<x,(0.4a,0.5b),(0.6a,0.5b)>$ is an IF$\pi\beta$GCS in (X,τ) but not an IFGPCS in (X, τ) .

Theorem 3.21:

Every *intuitionistc fuzzy generalized semi closed set* (IFGSCS in short) in (X, τ) is an IF$\pi\beta$GCS in (X,) but not conversely.

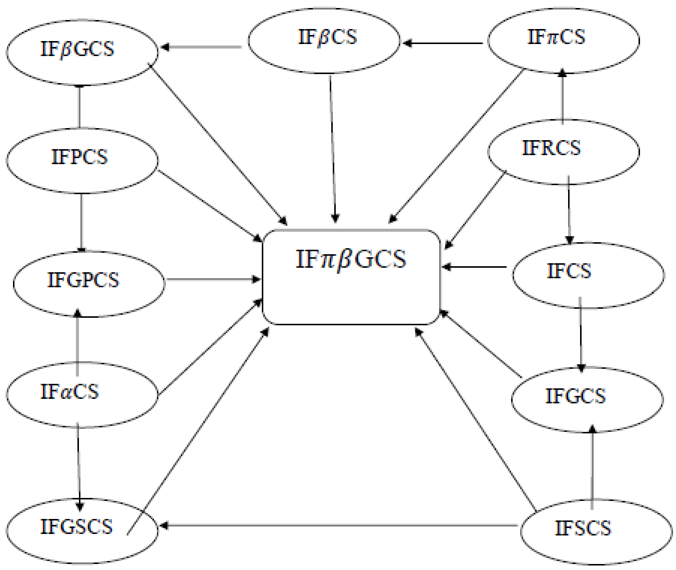
Proof:

Let A be an IFGSCS in X. Let A ⊆ U and U be an IFπOS in (X, τ). Therefore scl(A) = A ∪ int(cl(A)) ⊆ U, by hypothesis. This implies int(cl(A)) ⊆ U. Now int(cl(int(A)) ⊆ cl(int(A)) ∩ U ⊆ cl(A) ∩ U ⊆ cl(U) ∩ U ⊆ U. Hence A is an IF$\pi\beta$GCS in (X, τ).

Example 3.22:

Let X ={a,b} and let τ $=\{0\sim,G1,G2,1\sim\}$is an IFT on X, where $G1=<x,(0.5a,0.3b),(0.5a,0.7b)>$ and $G2=<x,(0.4a,0.3b),(0.6a,0.7b)>$. Then the IFS A $=<x,(0.3a,0.2b),(0.7a,0.8b)>$ is an IF$\pi\beta$GCS in (X,τ) but not an IFGSCS in (X, τ) as scl(A)= A∪ int(cl(A)) =A∪ G1= G1 ⊈ G2, but A ⊆ G2.

In the following diagram, we have provided relationship between various types of intuitionistic fuzzy closed sets.



Remark 3.23:

The intersection of any two IF$\pi\beta$GCS need not be an IF$\pi\beta$GCS in (X, τ) in general.

Example 3.24:

Let X={a,b}, G1=<x,(0.4,0.2),(0.6,0.8)> and G2 =<x, (0.4,0.4),(0.5,0.5)>. Then τ = {0~,G1, G2, 1~} is an IFT on X. Here the IFSs A=<x,(0.4,0.5),(0.5,0.4)> and B=<x, (0.5,0.2),(04,0.6)> are IF$\pi\beta$GCS in (X,τ) but A ∩ B =<x,(0.4,0.2),(0.5,0.6)> is not an IF$\pi\beta$GCS in (X,τ).

Theorem 3.25:

Let (X, τ) be an IFTS. Then for every A ϵ IF$\pi\beta$GC(X) and for every B ϵ IFS(X), A ⊆B ⊆ (A) implies B ϵ IF$\pi\beta$GC(X)

Proof:

Let B ⊆ U and U be an IFπOS. Since A ⊆B, A ⊆U, by hypothesis B⊆ (A). Therefore (B)⊆ $\beta cl(\beta cl(A))$= $\beta cl(A)$⊆ U. since A is an IF$\pi\beta$GCS. Hence B ϵ IFβGC(X).

Theorem 3.26:

If A is an IFβOS and IF$\pi\beta$GCS in (X, τ) then A is an IFβCS in (X, τ).

Proof:

Since A⊆ A and A is an IFβOS ,by hypothesis β(cl(A)) ⊆ A. But A ⊆ (cl(A). Therefore (cl(A))= A. Hence A is an IFβCS in (X, τ).

Theorem 3.27:

Let F ⊆ A ⊆ X where A is an IFβOS and an IF$\pi\beta$GCS in X. Then F is an IF$\pi\beta$GCS in A if and only if F is an IF$\pi\beta$GCS in (X, τ).

Proof:

Necessity:

Let U be an IFπOS in X and F ⊆ U. Also let F be an IF$\pi\beta$GCS in A. Then clearly F ⊆ A ∩ U and A ∩ U is an IFπOS in A. Hence (clA(F))⊆ A ∩ U and by theorem 2.1.24 , A is an IFβCS. Therefore (cl(A))= A. Now βcl(F)⊆ βcl(F)∩βcl(A)= βcl(F) ∩ A =βclA(F) ⊆A∩ U.That is βcl(F)⊆ U, whenever F ⊆ U. Hence F is an IF$\pi\beta$GCS in (X,).

Sufficiency:

Let V be an IFβOS in A such that F⊆V. Since A is an IFβOS in X, V is an IFβOS in X. Therefore βcl(F) ⊆V as F is an IF$\pi\beta$GCS in (X,τ). Thus, βclA(F) = βcl(F)∩ A ⊆ V ∩ A ⊆ V. Hence F is an IFβGCS in A.

Theorem 3.28:

Let A ⊆Y ⊆X and suppose that A is an IF$\pi\beta$GCS in X then A is an IF$\pi\beta$GCS relative to Y.

Proof:

Given that A ⊆Y ⊆ X and A is an IF$\pi\beta$GCS in X. Now let A ⊆Y ∩ U where U is an IFπOS in X. Since A is an IF$\pi\gamma$GCS in X, A ⊆ U implies βycl(A) ⊆ U. It follows that Y∩βcl(A)= βcl(A) ⊆ Y ∩ U = U. Thus A is an IF$\pi\beta$GCS relative to Y.

Theorem 3.29:

If an IFS A of an IFTS (X, τ) is an intuitionistic fuzzy nowhere dense then A is an IF$\pi\beta$GCS in X.

Proof:

If A is an intuitionistic fuzzy nowhere dense, then by definition int(cl(A))= 0~. Let A ⊆ U where U is an IFπOS in X. The βcl(A) =0~ ⊆ U and hence A is an IF$\pi\beta$GCS in X.

Theorem 3.30:

For an IFS A, the following conditions are equivalent:

i (i) A is an IFOS and an IF$\pi\beta$GCS

ii (ii) A is an IFROS.

Proof:

(i) ⟹ (ii) Let A be an IFOS and an IF$\pi\beta$GCS. Then βcl(A) ⊆ A and A ⊆ βcl(A). This implies that βcl(A)= A. Therefore A is an IFβCS, Since int(cl(int(A))) ⊆ A. Since A is an IFOS , int(A)=A. Therefore int(cl(A))=A .Since A is an IFOS and IFPOS. Hence A ⊆ int(cl(A)). Therefore A=int(cl(A)). Hence A is an IFROS.

(ii) ⟹ (i) Let a be an IFROS .Therefore A=int(cl(A)). Since every IFROS is an IFOS and A⊆ A . This implies int(cl(A)) ⊆ A. That is int(cl(int(A))) ⊆ A. Therefore A is an IFβCS. Hence A is an IF$\pi\beta$CS.

Theorem 3.31:

If A is both an IFαOS and an IF$\pi\beta$GCS in (X,). Then A is an IFβCS in (X, τ).

Proof:

Let A be an IFαOS .Then A is an IFβOS. As A⊆ A, by hypothesis βcl(A) ⊆ A ⊆ βcl(A), A is an IFβCS in (X,τ).

4. Intuitionistic fuzzy $\pi\beta$ generalized open sets

In this section we have introduced intuitionistic fuzzy $\pi\beta$generalized open sets and studied some of the properties.

Definition 4.1:

An IFS A is said to be an intuitionistic fuzzy $\pi\beta$generalized open sets (IF$\pi\beta G$OS in short) in (X, τ) if the complement Ac is an IF$\pi\beta$GOS in X.

The family of all IF$\pi\beta$GOSs of an IFTS (X, τ) is denoted by IF$\pi\beta$GO(X).

Example 4.2:

Let X={a,b} and τ = {0 ~ ,G1,G2,1~ } where G1=<x,(0.4,0.2),(0.6,0.8)> G2= <x.(0.7,0.8),(0.3,0.2)>.Then IFS A= <x,(0.5,0.5),(0.5,0.5)> is an IF$\pi\beta$GOS in (X , τ).

Theorem 4.3:

For any IFTS (X, τ) ,we have the following:

- Every IFOS in IF$\pi\beta$GOS in (X, τ) .
- Every IFαOS in IF$\pi\beta$GOS in (X, τ).
- Every IFROS in IF$\pi\beta$GOS in (X, τ).
- Every IFPOS in IF$\pi\beta$GOS in (X, τ).
- Every IFβOS in IF$\pi\beta$GOS in (X, τ).
- Every IF$\pi\beta$OS in IF$\pi\beta$GOSin (X, τ). But the converse are not true in general.

Proof: Straight forward.

Example 4.4:

Let X={a,b} and τ = {0 ~ ,G1,G2,1~ } where G1=<x,(0.4,0.2),(0.6,0.8)> G2= <x.(0.7a,0.8b),(0.3a,0.2b)>.Then IFS A= <x,(0.5a,0.5b),(0.5a,0.5b)> is an IF$\pi\beta$GOS in (X , τ),but not an IFOS in (X, τ) as cl(A) = G1 ≠ A.

Example 4.5:

Let X={a,b} and τ = {0 ~ ,G1,G2,1~ } where G1=<x,(0.4,0.2),(0.6,0.8)> G2= <x.(0.7a,0.8b),(0.3a,0.2b)>.Then IFS A= <x,(0.5a,0.5b),(0.5a,0.5b)> is an IF$\pi\beta$GOS in (X,τ), but not an IFαOS in (X, τ) as int(cl(int(A)))=int(cl(G1))=int(G1c)=G1 , A ⊈ G1.

Example 4.6:

Let X={a,b} and τ = {0 ~ ,G1,G2,1~ } where G1=<x,(0.4,0.2),(0.6,0.8)> G2= <x.(0.7a,0.8b),(0.3a,0.2b)>.Then IFS A= <x,(0.5a,0.5b),(0.5a,0.5b)> is an IF$\pi\beta$GOS in (X , τ),but not an IFROS in (X, τ) as int(cl(A))=int(G1c)=G1,≠A.

Example 4.7:

Let X={a,b} and τ = {0 ~ ,G1,G2,1~ } where G1=<x,(0.4,0.2),(0.6,0.8)> G2= <x.(0.7a,0.8b),(0.3a,0.2b)>.Then IFS A= <x.(0.5a,0.5b),(0.5a,0.5b)> is an IF$\pi\beta$GOS in (X , τ),but not an IFPOS in (X, τ) as int(cl(A))=int(G1c)=G1 ,A ⊈ G1.

Example 4.8:

Let X={a,b} and τ = {0 ~ ,G1,G2,1~ } where G1=<x,(0.4,0.2),(0.6,0.8)> G2= <x.(0.7a,0.8b),(0.3a,0.2b)>.Then IFS A= <x,(0.5a,0.5b),(0.5a,0.5b)> is an

IF$\pi\beta$GOS in (X,τ),but not an IFβOS in (X,τ) as cl(int(cl(A)))=cl(int(G1c)=cl(G1) =G1c ,A $\nsubseteq$ G1.

Example 4.9:

Let X={a,b} and τ = {0 ~ ,G1,G2,1~ } where G1=<x,(0.4,0.2),(0.6,0.8)> G2= <x.(0.7a,0.8b),(0.3a,0.2b)>.Then IFS A= <x,(0.5a,0.5b),(0.5a,0.5b)> is an IF$\pi\beta$GOS in (X , τ),but not an IFπOS in (X, τ) as int(cl(A))=int(G1c)=G1, A$\neq$G1.

Theorem 4.10:

Let (X, τ) be an IFTS. Then for every A ϵ IF$\pi\beta$GO(X) and for every B ϵ IFS(X), β $int(A)$⊆ B ⊆A ⇒ B ϵ IF$\pi\beta$GO(X).

Proof:

Let A be an IF$\pi\beta$GOS of X and B be any IFS on X. Let β $int(A)$⊆B ⊆A. Then Ac is an IFπβGCS and Ac ⊆ Bc⊆ β (Ac). Therefore Bc is an IFπβGCS which implies B is an IFπβGOS in X. Hence B ϵ IFπβGO(X).

Theorem 4.11:

If A is an IFRCS and B is an IFβOS, then A∪B is an IFπβGOS in (X, τ).

Proof:

Let B be an IFβOS and A be an IFRCS. Then B⊆$cl(int(cl(B)))$ and $cl(int(A))$= A .Therefore A∪B ⊆ A ⊆ cl(int(cl(B)))= cl(int(A)) ∪cl(int(cl(B))) ⊆ cl(int(cl(A)))∪ cl(int(cl(B)))=cl(int(cl(A)) ∪ int(cl(B))) ⊆ cl(int(cl(A) ∪ cl(B)) . Therefore A∪B is an IFβOS and hence by theorem 2.2.3, A∪B is an IF$\pi\beta$GOS in X.

Theorem 4.12:

If an IFS A of an IFTS in both an IFCS and an IFGOS, then A is an IF$\pi\beta$GOS in (X, τ).

Proof:

Suppose A is both an IFCS and IFGOS. Then as A ⊆ A, by hypothesis A ⊆ int(A). But int(A) ⊆ A. Therefore int(A) = A. We have A is an IFπOS, since every IFπOS is an IF$\pi\beta$GOS. Hence A is an IF$\pi\beta$GOS in X.

Theorem 4.13:

Let (X, τ) be an IFTS. Then for every A ϵ IFS(X) and for every B ϵ IFβO(X), B ⊆ A ⊆ int(cl(int(B))) ⟹ Aϵ IFπβGO(X).

Proof:

Let B be an IFβOS. Then B ⊆ cl(int(cl(B))). By hypothesis A ⊆ int(cl(int(B))) ⊆ int(cl(int(cl(int(cl(B)))))⊆ int(cl(cl(int(cl(B)))))= int(cl(int(cl(B)))) ⊆ int(cl(cl(A))) ⊆int(cl(A)) as B ⊆ A. Therefore A is an IFPOS. By theorem 2.2.3, A is an IFβGOS . Hence A ϵIFπβGO(X).

Theorem 4.14:

If A is an IFβCS and an IF$\pi\beta$GOS in (X, τ), then A is an IFβOS in (X, τ).

Proof:

As A⊇A ,by hypothesis βint(A) ⊇A. But we have A⊇ βint(A). This impiles A= βint(A). Hence A is an IFβOS in (X, τ).

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ON INTUITIONISTIC FUZZY $\pi\gamma$ GENERALIZED CONTINUOUS MAPPINGS

Prema S* and Archana A

Department of Mathematics

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore, Tamilnadu, India

*Correspondiing author E-mail: premaparameswaran14@gmail.com

Abstract:

This chapter is devoted to the study of intuitionistic fuzzy topological spaces. In this chapter $\pi\gamma$ generalized continuous mappings in intuitionistic fuzzy topological spaces is introduced. Also, we have analyzed some properties of π γ generalized continuous mappings in intuitionistic fuzzy topological spaces.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy $\pi\gamma$ generalized continuous mappings.

1. Introduction:

In 1965, the concept of Fuzzy sets was introduced by Lofti A. Zadeh [10] and in 1968, Chang [3] introduced and developed fuzzy topology. After the introduction of fuzzy set and fuzzy topology, several authors conducted researchers on the generalization of these notions. In the year 1986, the notion of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets and Coker [4] introduced the concept of intuitionistic fuzzy topological spaces in 1997. In 2017, Prema S and Jayanthi D [9] has introduced intuitionistic fuzzy γ generalized continuous mappings. In this chapter we have introduced $\pi\gamma$ generalized continuous mappings in intuitionistic fuzzy topological spaces and investigated some of their properties and obtained some interesting characteristics.

2. Preliminaries:

Definition 2.1: [1]

Let X be a non-empty fixed set. An ***intuitionistic fuzzy set*** (IFS in short) A in X is an object having the form A = { $<x,\mu A(x),\nu A(x)>/x \in X\}$ where the functions $\mu_A(x)$: X $\rightarrow$[0,1] and $\nu_A(x)$:X$\rightarrow$[0,1] denotes the degree of membership (namely $\mu_A(x)$) and the degree of non – membership (namely $\nu_A(x)$) of each element x $\in X$ to the set A, respectively, and $0 \leq \mu_A(x)+\nu_A(x) \leq 1$ for each x $\in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1]

Let A and B be IFSs of the form A= $\{<x,(x),\nu_A(x)>/x \in X\}$ and B= $\{<x,\mu_B(x),\nu_B(x)>/x \in X\}$. Then

a) A ⊆ B if and only if $\mu_A(x)\leq\mu_B(x)$ and $\nu_A(x)\geq\nu_B(x)$ for all x $\in X$

b) A=B if and only if A ⊆ B and B ⊆ A

c) Ac = $\{<x,\nu_A(x),\mu_A(x)>/x \in X\}$

d) A ∩ B = $\{<x,\mu_A(x)\wedge\mu_B(x),\nu_A(x)\vee \nu_B(x)>/x \in X\}$

e) A ∪ B = $\{<x,\mu_A(x)\vee\mu_B(x),\nu_A(x)\wedge \nu_B(x)>/x \in X\}$

For the sake of simplicity, we shall use the notation A= $< x, (\mu_A,),(\nu_A,\nu_B)>$ instead of A= $<x,(A/\mu_A,B/\mu_B),(A/\nu_A,B/\nu_B)>$.

The intuitionistic fuzzy sets 0∼ = $\{<x,0,1)>/x \in X\}$ and 1∼ = $\{<x,0,1)>/x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [4]

An ***intuitionistic fuzzy topology*** (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

i. 0~, 1~ $\in$ τ

ii. G1 ∩ G2 $\in$ τ, for any G1, G2 $\in$ τ

iii. ∪ $G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an ***intuitionistic fuzzy topological space*** (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [6]

Let A be an IFS in an IFTS in (X,τ). Then the intuitionistic fuzzy γ- interior and intuitionistic fuzzy γ-closure of A are defined by

i. γint(A)= ∪ {G/G is an IFγOS in X and G⊆ A},

ii. γcl(A) = ∩{K/K is an IFγCS in X and A⊆K}.

Note that for any IFS A in (X,τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A^c)= (\gamma cl(A))^c$.

Definition 2.5: [8]

Let A be an IFS in (X,τ), then

i. γint(A)⊆ A∩((cl(int(A)) ∩int(cl(A)))

ii. γcl(A)⊇ A ∪((cl(int(A)) ∩int(cl(A)))

Definition 2.6: [8]

An IFS A is an IFTS (X,τ) is said to be an ***intuitionistic fuzzy γ generalized closed set*** (IFγGCS for short) if γcl(A) ⊆ U whenever A ⊆ U and U is an IFγOS in (X,τ).

Definition 2.7:[2]

An IFS A in (X,) is said to be a *intuitionistic fuzzy πγ generalized closed sets* (IFπγGCS in short) if γcl(A)⊆ U whenever A ⊆ U and U is an $IF\pi OS$ in (X, τ).

Definition 2.8:[5]

Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an **intuitionistic fuzzy continuous** (IF continuous) mapping if f -1 (V) is an IFCS in(X,) for every IFCS V of (Y,)

Definition 2.9:[7]

Let f be a mapping from an IFTS (X,) into an IFTS (Y,). Then f is said to be an

i. ***intuitionistic fuzzy semi continuous*** (IFS continuous) ***mapping*** if f^{-1} (V) is an IFSCS in (X, τ) for every IFCS V of (Y, σ),

ii. ***intuitionistic fuzzy α*** **continuous** (IFα continuous) ***mapping*** if f^{-1} (V) is an IFαCS in (X, τ) for every IFCS V of (Y, σ),

iii. ***intuitionistic fuzzy pre continuous*** (IFP continuous) ***mapping*** if f^{-1} (V) is an IFPCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.10:[6]

Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an ***intuitionistic fuzzy γ continuous*** (IFγ continuous) mapping if f^{-1} (V) is an IFγCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.11:[9]

A mapping f: (x, τ) ⟶ (y,σ) is called an ***intuitionistic fuzzy γ generalized continuous*** (IFγG continuous) mapping if f^{-1} (V) is an IFγGCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.12: [8]

An intuitionistic fuzzy point (IFP in short) written as (α,) is defined to be an IFS of X given by

$$p_{(\alpha,\beta)} = \begin{cases} (\alpha,\beta) & \text{if } x = p \\ (0,1) & otherwsie \end{cases}$$

An intuitionistic fuzzy point $_{(\alpha,)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\geq \nu_A$.

3. INTUITIONISTIC FUZZY π γ GENERALIZED CONTINUOUS MAPPINGS

In this section we have introduced intuitionistic fuzzy $\pi\gamma$ generalized continuous mappings and examined some of the properties.

Definition 3.1:

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is called an intuitionistic fuzzy π γ generalized continuous (IF$\pi\gamma$G continuous for short) mappings if f^{-1} (V) is an IF$\pi\gamma$GCS in (X,τ) for every IFCS V of (Y,σ).

For the sake of simplicity, we shall use the notation A= < $x,(\mu_A,\mu_B),(\nu_A,\nu_B)$> instead of A= <$x,(a/\mu_A,b/\mu_B),(a/\nu_A,b/\nu_B)$> in the following examples. Similarly, we shall use the notation B= < $y,(\mu_u,\mu_v),(\nu_u,\nu_v)$> instead of B= <$y,(a/\mu_u,b/\mu_v),(a/\nu_u,b/\nu_v)$> in the following examples.

The intuitionistic fuzzy sets 0~ = {<x,0,1)>/$x \in X$} and 1~= {<x,0,1)>/$x \in X$} are respectively the empty set and the whole set of X.

Example 3.2 :

Let X={a,b}, Y={u,v} and G1= <x, $(0.4_a,0.2_b),(0.6_a,0.8_b)$>, G2=<x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$>,$G_3$=<y,$(0.5_u,0.6_v),(0.5_u,0.4_v)$>. Then τ = {0~,$_1$,G_2,1~} and σ={0~,G_3,1~} are IFTs on X and Y respectively. Define a mapping f : $(X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS G_3^c =<y,$(0.5_u,0.4_v),(0.5_u,0.6_v)$> is an IFCS in Y.

Then $f^{-1}(G_3^c)$ = <x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,). Therefore, f is an IF$\pi\gamma$G continuous mapping.

Theorem 3.3:

Every IF continuous mapping is an IF$\pi\gamma$G continuous mapping in (X,) but not conversely in general.

Proof:

Let f: (X,) → (Y,) be an IF continuous mapping. Let V be an IFCS in Y. Then f^{-1} (V) is an IFCS in X. Since every IFCS is an IF$\pi\gamma$GCS, f^{-1} (V) is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.4:

Let X={a,b}, Y={u,v} and G_1=<x,$(0.4_a,0.2_b),(0.6_a,0.8_b)$>, G_2 =<x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$>,$G_3$=<y,$(0.5_u,0.6_v),(0.5_u,0.4_v)$>. Then τ= {0~,$_1$,G_2,1~} and σ={0~,G_3,1~} are IFTs on X and Y respectively. Define a mapping f : $(X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS (G_3^c) = <y,$(0.5_u,0.4_v),(0.5_u,0.6_v)$> is an IFCS in Y.

Then $f^{-1}(G_3^c)$ = <x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFCS in X, as $cl(f^{-1}(G_3^c)) = G^2{}_c \neq f^{-1}(G_3^c)$, f is not an IF continuous mapping.

Theorem 3.5:

Every IFS continuous mapping is an IF$\pi\gamma$G continuous mapping in (X,) but not conversely in general.

Proof:

Let f: (X,) → (Y,) be an IFS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IF $\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.6:

Let X={a,b}, Y={u,v} and G_1 = <x,(0.5_a,0.6_b),(0.5_a,0.4_b)>, G_2=<x,(0.4_a,0.3_b),(0.6_a,0.7_b)>,G_3=<y,(0.7_u,0.8_v),(0.3_u,0.2_v)>. Then τ ={0~,G_1,G_2,1~} and σ={0~,G_3,1~} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS $G^3{}_c$ = <y, (0.3_u,0.2_v),(0.7_u,0.8_v)> is an IFCS in Y. Then f-1($G^3{}_c$) = <x, (0.3_a,0.2_b),(0.7_a,0.8_b)> is an IFS in X. Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3^c)) = int(G_1^c)$ = G1 $\nsubseteq$ f-1(G_3^c), f is not an IFS continuous mapping.

Theorem 3.7:

Every IFP continuous mapping is an IF$\pi\gamma$G continuous mapping in (X,) but not conversely in general.

Proof:

Let f: (X,) → (Y,) be an IFP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.8:

Let X={a,b} , Y={u,v} and G1 = <x, (0.4_a,0.2_b),(0.6_a,0.8_b)>, G_2=<x,(0.5_a,0.4_b),(0.5_a,0.6_b)>,G_3=<y,(0.5_u,0.6_v),(0.5_u,0.4_v)>. Then τ ={0~,G_1,G_2,1~} and σ={0~,G_3,1~} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y,(0.5_u,0.4_v),(0.5_u,0.6_v)> is an IFCS in Y.

Then $f^{-1}(G_3^c)$ = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3^c))) = cl(G_2)=G_2^c \not\subseteq f^{-1}(G_3^c)$, f is not an IFP continuous mapping.

Theorem 3.9:

Every IFR continuous mapping is an IF$\pi\gamma$G continuous mapping in (X,) but not conversely in general.

Proof:

Let f: (X,) → (Y,) be an IFR continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.10:

Let X={a,b} , Y={u,v} and $G_1 = <x, (0.4_a,0.2_b),(0.6_a,0.8_b)>$, $G_2=<x,(0.5_a,0.4_b),(0.5_a,0.6_b)>$, $G_3=<y,(0.5_u,0.6_v),(0.5_u,0.4_v)>$. Then $\tau =\{0\sim,G_1,G_2,1\sim\}$ and $\sigma=\{0\sim,G_3,1\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS $G_3^c = <y,(0.5_u,0.4_v),(0.5_u,0.6_v)>$ is an IFCS in Y.

Then $f^{-1}(G_3^c) = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>$ is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,). Therefore f is an IF$\pi\gamma$G continuous mapping but since f-1(G3c) is not an IFRCS in X, as $cl(int(f^{-1}(G_3^c))) = cl(G_2)=G_2^c \neq f^{-1}(G_3^c)$, f is not an IFR continuous mapping.

Theorem 3.11:

Every IFα continuous mapping is an IF$\pi\gamma$G continous mapping in (X,) but not conversely in general.

Proof:

Let f: (X,) → (Y,) be an IFα continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.12:

Let X={a,b} , Y={u,v} and $G_1 = <x, (0.4_a,0.2_b),(0.6_a,0.8_b)>$, $G_2=<x,(0.5_a,0.4_b),(0.5_a,0.6_b)>$, $G_3=<y,(0.5_u,0.6_v),(0.5_u,0.4_v)>$. Then $\tau =\{0\sim,G_1,G_2,1\sim\}$ and $\sigma=\{0\sim,G_3,1\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS $G_3^c = <y,(0.5_u,0.4_v),(0.5_u,0.6_v)>$ is an IFCS in Y.

Then $f^{-1}(G_3^c) = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>$ is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,). Therefore f is an IF$\pi\gamma$G continuous mapping but since f-1(G3c) is not an IFαCS in X, as $cl(int(f^{-1}(G_3^c))) = cl(G_2)=G_2^c \neq f^{-1}(G_3^c)$, f is not an IFR continuous mapping.

Theorem 3.13:

Every IFπ continuous mapping is an IF$\pi\gamma$G continous mapping in (X,) but not conversely in general.

Proof:

Let f: (X,) → (Y,) be an IFπ continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFπCS in X. Since every IFπCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.14:

Let X={a,b}, Y={u,v} and G_1 = <x,($0.5_a,0.3_b$),($0.5_a,0.7_b$)>, G_2=<x,($0.4_a,0.3_b$),($0.6_a,0.7_b$)>,G_3=<y,($0.7_u,0.8_v$),($0.3_u,0.2_v$)>. Then τ ={0~,G_1,G_2,1~} and σ={0~,G_3,1~} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS $G^3{}_c$ = <y, ($0.3_u,0.2_v$),($0.7_u,0.8_v$)> is an IFCS in Y. Then f-1($G^3{}_c$) = <x, ($0.3_a,0.2_b$),($0.7_a,0.8_b$)> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,). Therefore f is an IF$\pi\gamma$G continuous mapping but not an IFπ continuous mapping, since $f^{-1}(G_3^c)$ is not an IFπCS in X, as $cl(int(f^{-1}(G_3^c))= 0\sim \neq$ f-1(G_3^c).

Theorem 3.15:

Every IFγ continuous mapping is an IF$\pi\gamma$G continous mapping in (X,) but not conversely in general.

Proof:

Let f: (X,) → (Y,) be an IFγ continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFγCS in X. Since every IFγCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.16:

Let X={a,b}, Y={u,v} and G_1 = <x,($0.5_a,0.4_b$),($0.5_a,0.6_b$)>, G_2=<x,($0.4_a,0.3_b$),($0.6_a,0.7_b$)>,G_3=<y,($0.5_u,0.6_v$),($0.4_u,0.4_v$)>. Then τ ={0~,G_1,G_2,1~} and σ={0~,G_3,1~} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS $G^3{}_c$ = <y, ($0.4_u,0.4_v$),($0.5_u,0.6_v$)> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,($0.4_a,0.4_b$),($0.5_a,0.6_b$)> is an IFS in X. Hence f-1(G3c) is an IF$\pi\gamma$GCS in (X,). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3^c))) \cap$

$cl(int(f^{-1}(G_3^c))) = int(G_1^c) \cap cl(G_2) = G1 \cap G_1^c \not\subseteq f^{-1}(G_3^c)$, $f^{-1}(G_3^c)$ is not an IFS continuous mapping.

Theorem 3.17:

Every IFG continuous mapping is an IF$\pi\gamma$G continous mapping in (X,) but not conversely in general.

Proof:

Let f: (X,) → (Y,) be an IFG continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGCS in X. Since every IFGCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.18:

Let X={a,b} , Y={u,v} and G_1 = <x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$>, G_2=<x,$(0.4_a,0.3_b),(0.6_a,0.7_b)$>,$G_3$=<y,$(0.5_u,0.6_v),(0.4_u,0.4_v)$>.Then $\tau = \{0\sim,G_1,G_2,1\sim\}$ and $\sigma = \{0\sim,G_3,1\sim\}$ are IFTs on X and Y respectively. Define a mapping f : $(X,\tau) \to (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS G_3^c = <y, $(0.4_u,0.4_v),(0.5_u,0.6_v)$> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,$(0.4_a,0.4_b),(0.5_a,0.6_b)$> is an IFS in X. Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFGCS in X, as $cl(f^{-1}(G_3^c)) = G_1^c \not\subseteq G$, $f^{-1}(G_3^c)$ is not an IFG continuous mapping.

Theorem 3.19:

Every IFGS continuous mapping is an IF$\pi\gamma$G continous mapping in (X,) but not conversely in general.

Proof:

Let f: (X,) → (Y,) be an IFGS continuous mapping. Let V be an IFCS in Y. Then $f^1(V)$ is an IFGSCS in X. Since every IFGSCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.20:

Let X={a,b} , Y={u,v} and G_1 = <x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$>, G2=<x,$(0.4_a,0.3_b),(0.6_a,0.7_b)$>,$G_3$=<y,$(0.5_u,0.6_v),(0.4_u,0.4_v)$>.Then $\tau = \{0\sim,G_1,G_2,1\sim\}$ and $\sigma = \{0\sim,G_3,1\sim\}$ are IFTs on X and Y respectively. Define a mapping f : $(X,\tau) \to (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS G_3^c = <y,$(0.4_u,0.4_v),(0.5_u,0.6_v)$> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,$(0.4_a,0.4_b),(0.5_a,0.6_b)$> is an IFS in X. Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFGSCS in X, as $f^{-1}(G_3^c) \cup int(cl(f^{-1}(G_3^c))) = f^{-1}(G_3^c) \cup int(G_1^c) = f^{-1}(G_3^c) \cup G1 = G1 \not\subseteq G2$, $f^{-1}(G_3^c)$ is not an IFGS continuous mapping.

Theorem 3.21:

Every IFGP continuous mapping is an IF$\pi\gamma$G continous mapping in (X,) but not conversely in general.

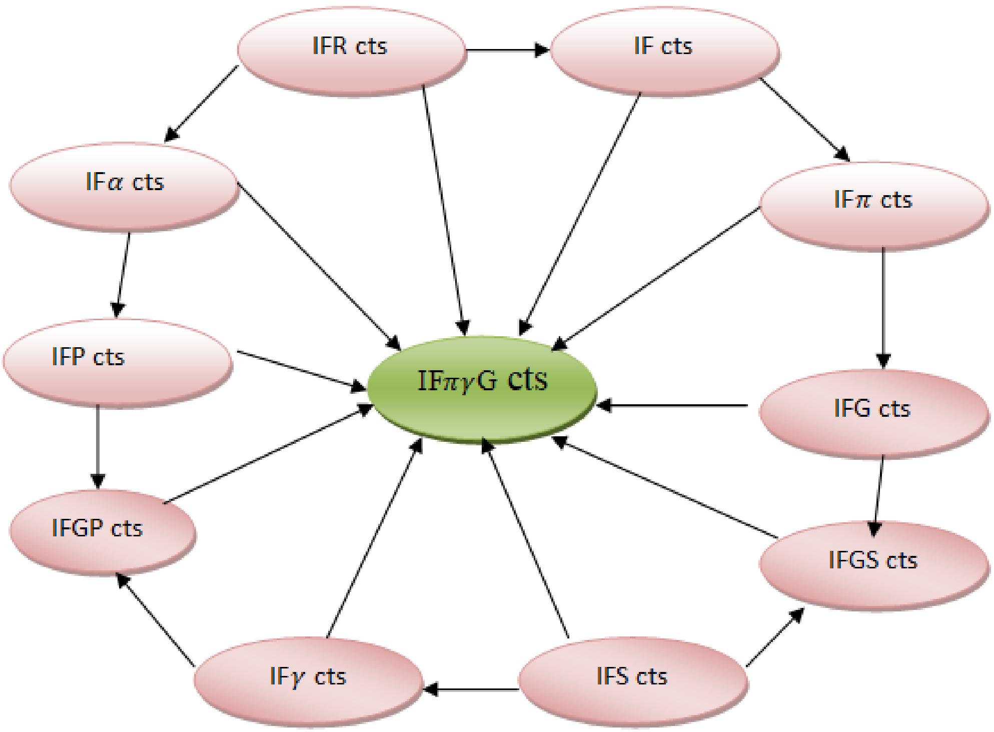
Proof:

Let f: (X,) → (Y,) be an IFGS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGPCS in X. Since every IFGPCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.22:

Let X={a,b} , Y={u,v} and G_1 = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>, G_2=<x,(0.4_a,0.3_b),(0.6_a,0.7_b)>,G_3=<y,(0.5_u,0.6_v),(0.4_u,0.4_v)>.Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y,(0.4_u,0.4_v),(0.5_u,0.6_v)> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,(0.4_a,0.4_b),(0.5_a,0.6_b)> is an IFS in X. Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFGPCS in X, $f^{-1}(G_3^c)$ is not an IFGP continuous mapping.

The relationship between various types of intuitionistic fuzzy continuity is given in the following figure. In this figure 'cts' means continuous.



Theorem 3.23

A mapping f: (X,) → (Y,) is an IF$\pi\gamma$G continuous mapping if and only if the inverse image of each IFπOS in Y is an IF$\pi\gamma$GOS in (X,).

Proof:

Necessity:

Let A be an IFπOS in Y. This implies Ac is an IFπCS in Y. Then f^{-1} (Ac) is an IF$\pi\gamma$GCS in X, by hypothesis. Since f^{-1}(Ac) = (f^{-1}(A)c), f^{-1}(A) is an IF$\pi\gamma$GOS in X.

Sufficiency:

Let A be an IFπCS in Y. Then Ac is an IFπOS in Y. By hypothesis f^{-1}(Ac) is an IF$\pi\gamma$GOS in X. Since f^{-1}(Ac) = (f^{-1}(A)c), (f-1(A)c) is an IF$\pi\gamma$GOS in X. Therefore f^{-1}(A) is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Theorem 3.24:

If f : (X,τ) → (Y,σ) is an IF$\pi\gamma$G continous mapping then for each IFP $p_{(\alpha,\beta)}$of X and each A ϵ σ such that f($p_{(\alpha,\beta)}$) q A, there exists an IF$\pi\gamma$GOS B of X such that $p_{(\alpha,\beta)}$ q B and f(B) ⊆ A.

Proof:

Let (α,) be an IFP of X and A ϵ σ such that f($p_{(\alpha,\beta)}$)q A. Put B= f^{-1}(A). Then by hypothesis, B is an IF$\pi\gamma$GOS in X such that $_{(\alpha,)}$q B and f(B) =f(f^{-1}(A)) ⊆ A.

Theorem 3.25:

A mapping f : (X,τ) → (Y,σ) is an IF$\pi\gamma$G continous mapping if cl(int(cl(f^{-1}(A)))) ⊆ f^{-1}(cl(A)) for every IFS A in Y.

Proof:

Let A be an IFπOS in Y then A^c is an IFπCS in Y. By hypothesis, cl(int(cl(f^{-1}(Ac)))) ⊆ f^{-1}(cl(Ac))= f^{-1}(A^c). Now (int(cl(int(f^{-1}(A)))))^c = cl(int(cl(f^{-1}(Ac)))) ⊆ f^{-1}(Ac) = f^{-1}(cl(A))^c. This implies that f^{-1}(A) ⊆ (int(cl(int(f^{-1}(A))))). Hence f^{-1}(A) is an IFαOS and hence it is an IF$\pi\gamma$GOS. Therefore f is an IF$\pi\gamma$G continuous mapping.

Theorem 3.26:

Let f: (X,) → (Y,) be an IF$\pi\gamma$G continuous mapping and g: (Y,) → (Z,) is an IF continuous mapping then g ∘ f : (X,τ) → (Z,δ) is an IF$\pi\gamma$ continuous mapping.

Proof:

Let V be an IFCS in Z. Then g-1(V) is an IFCS in Y, by hypothesis. Since f is an IF$\pi\gamma$G continuous mapping, f^{-1}(g^{-1}(V)) is an IF$\pi\gamma$GCS in X. Hence g ∘ f is an IF$\pi\gamma$G continuous mapping.

Theorem 3.27:

Let f: (X,) → (Y,) be an mapping from an IFTS X into an IFTS Y that satisfies $f^{-1}(int(B)) = int(cl(f^{-1}(B)))$ for every IFS B in Y. Then f is an IF$\pi\gamma$G continuous mapping.

Proof:

Let B be an IFπOS in Y. Then int(cl(B))= B, by hypothesis $f^{-1}(B) = int(cl(f^{-1}(B)))$. This implies $f^{-1}(B)$ is an IFROS in X. Therefore it is an IF$\pi\gamma$GOS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

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RATIO TYPE ESTIMATORS FOR FINITE POPULATION MEAN USING KNOWN PARAMETERS OF AUXILIARY VARIABLE

Rajesh Tailor[1], Med Ram Verma*[2], Sunil Chouhan[1] and Ritesh Tailor[3]

[1]School of Studies in Statistics, Vikram University, Ujjain-456010 (M.P.)

[2]Division of LES &IT, ICAR-IVRI, Izatnagar, Bareilly, UP

[3]Institute of Wood Science and Technology (IWST) Bangalore, India

*Corresponding author E-mail: medramverma@rediffmail.com

Abstract:

This chapter discusses the problem of estimation of finite population mean in stratified random sampling. In fact, in this chapter two ratio type estimators of population mean have been proposed using known parameters of auxiliary variable. Biases and mean squared errors of proposed estimators have been obtained upto the first degree of approximation. The suggested estimators have been compared with usual unbiased estimator, combined ratio estimator and estimators given by Kadilar and Cingi (2003). An empirical study has been carried out to demonstrate the performance of the proposed estimators.

1. Introduction:

Many times, the information on a variable x closely related to the study variable y is easily available or it can be collected at very cheap cost. For example, in estimating the total production of any crop, information on production of the same crop for previous year may be available for all units of the population. This previous year production of a crop can be considered as an auxiliary variable (x). In this situation, estimator for population mean ($\bar{Y}$) of study variabley based on information on x would be more efficient than the estimator based on information only on the study variable y.

Use of auxiliary information has been in practice for improving the efficiency of the estimators. The basic concept behind the use of auxiliary information is that the correlation coefficient between the study variable and auxiliary variable helps in improving the efficiency of the estimators of parameters of the study variable. Cochran (1940) envisaged a ratio method of estimation that provides classical ratio estimator for population mean. Ratio estimator given by Cochran (1940) has better efficiency as compared to the simple mean estimator when the study variable and auxiliary variable are positively correlated and the correlation coefficient is greater than half of the ratio of coefficient of variation of the auxiliary variable to the coefficient of variation of the study variable.

Major disadvantage of the ratio type estimators is that these do not perform better in terms of efficiency in case of negative correlation between the study variable and auxiliary variable.

For the case of negative correlation coefficient between the study variable and auxiliary variable, Robson (1957) suggested a product method of estimation that provides product estimator for population mean.

Many researchers used auxiliary information in the form of known parameters for the estimation of unknown parameters. Sisodiya and Dwivedi (1981) used coefficient of variation of the auxiliary variable. Singh and Upadhyaya (1999) utilized both coefficient of variation as well as coefficient of kurtosis. Singh and Tailor (2003) used correlation coefficient between the study variable and auxiliary variable for the estimation of population mean.

Hansen *et al.* (1946) developed combined ratio estimator using auxiliary information at estimation stage in stratified random sampling. Later Kadilar and Cingi (2003) utilized known parameters of auxiliary variable and developed many ratio type estimators in stratified random sampling. Singh *et al.* (2008) studied properties of Bahl and Tuteja (1991) ratio type estimator in stratified random sampling.

Consider a finite population U of size N consisting of units $U_1, U_2, ..., U_N$. Associated with the unit U_i, there are two real quantities (y_i, x_i), $i = 1, 2, ..., N$, representing the values of the study variable*y* and a positively correlated auxiliary variable*x*. Population U is divided into k homogeneous strata of size N_h (*h = 1,2…, k*). A sample of size n_h is drawn from each stratum following the simple random sampling without replacement method. Let $\bar{y}_{st} = \sum_{h=1}^{k} W_h \bar{y}_h$ and $\bar{x}_{st} = \sum_{h=1}^{k} W_h \bar{x}_h$ be the unbiased estimators of the population mean $\bar{Y}$ and $\bar{X}$ of the study variable and auxiliary variable, respectively, where

$W_h = (N_h / N)$: weight of h^{th} stratum,

$\bar{y}_h = (1/n_h)\sum_{j=1}^{n_h} y_{hj}$: sample mean of the study variable y for h^{th} stratum and

$\bar{x}_h = (1/n_h)\sum_{j=1}^{n_h} x_{hj}$: sample mean of the auxiliary variable x for h^{th} stratum.

Assuming that $\bar{X}$ is known, the combined ratio estimator for estimating the population mean $\bar{Y}$ is defined as

$$\hat{\bar{Y}}_{RC} = \bar{y}_{st}\left(\bar{X} / \bar{x}_{st}\right). \qquad (1.1)$$

The bias and mean squared error expressions of the combined ratio estimator $\hat{\bar{Y}}_{RC}$ upto the first degree of approximation are

$$Bias(\hat{\bar{Y}}_{RC}) = (1/\bar{X})\sum_{h=1}^{k} W_h^2 \gamma_h (RS_{xh}^2 - S_{yxh}), \qquad (1.2)$$

$$MSE(\hat{\bar{Y}}_{RC}) = \sum_{h=1}^{k} W_h^2 \gamma_h (S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh}). \qquad (1.3)$$

where

$$R = \bar{Y} / \bar{X}, \qquad \gamma_h = \left(\frac{N_h - n_h}{N_h n_h}\right), \qquad S_{yh}^2 = (1/(N_h - 1))\sum_{j=1}^{N_h} (y_{hj} - \bar{Y}_h)^2,$$

$$S_{xh}^2 = (1/(N_h - 1))\sum_{j=1}^{N_h} (x_{hj} - \bar{X}_h)^2 \text{ and } S_{yxh} = (1/(N_h - 1))\sum_{j=1}^{N_h} (y_{hj} - \bar{Y}_h)(x_{hj} - \bar{X}_h).$$

Sisodia and Dwivedi (1981) suggested a modified ratio estimator of $\bar{Y}$ using coefficient of variation of auxiliary variable *x* in simple random sampling as

$$\hat{\bar{Y}}_1 = \bar{y}\left[(\bar{X} + C_x)/(\bar{x} + C_x)\right]. \qquad (1.4)$$

Here, $(\bar{x}, \bar{y})$ are the sample means for (x, y) and C_x is the coefficient of variation present in auxiliary characteristic *x*.

Singh *et al.* (2004) proposed another ratio estimator for $\bar{Y}$, using the coefficient of kurtosis $\beta_2(x)$ of auxiliary variable*x* in simple random sampling as

$$\hat{\bar{Y}}_2 = \bar{y}\left[(\bar{X} + \beta_2(x))/(\bar{x} + \beta_2(x))\right]. \qquad (1.5)$$

Upadhyaya and Singh (1999) used coefficient of kurtosis and coefficient of variation of auxiliary variable and suggested estimators of population mean $\bar{Y}$ in simple random sampling as

$$\hat{\bar{Y}}_3 = \bar{y}\left[(\bar{X}\beta_2(x) + C_x)/(\bar{x}\beta_2(x) + C_x)\right] \text{ and} \qquad (1.6)$$

$$\hat{\bar{Y}}_4 = \bar{y}\left[(\bar{X}C_x + \beta_2(x))/(\bar{x}C_x + \beta_2(x))\right]. \qquad (1.7)$$

Kadilar and Cingi (2003) defined $\hat{\bar{Y}}_1$, $\hat{\bar{Y}}_2$, $\hat{\bar{Y}}_3$ and $\hat{\bar{Y}}_4$ in stratified random sampling as

$$\hat{\bar{Y}}_{st1} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h (\bar{X}_h + C_{xh}) / \sum_{h=1}^{k} W_h (\bar{x}_h + C_{xh})\right], \qquad (1.8)$$

$$\hat{\bar{Y}}_{st2} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h\,(\bar{X}_h + \beta_{2h}(x)) / \sum_{h=1}^{k} W_h\,(\bar{x}_h + \beta_{2h}(x))\right], \tag{1.9}$$

$$\hat{\bar{Y}}_{st3} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h\,(\bar{X}_h\beta_{2h}(x) + C_{xh}) / \sum_{h=1}^{k} W_h\,(\bar{x}_h\beta_{2h}(x) + C_{xh})\right], \tag{1.10}$$

$$\hat{\bar{Y}}_{st4} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h\,(\bar{X}_h C_{xh} + \beta_{2h}(x)) / \sum_{h=1}^{k} W_h\,(\bar{x}_h C_{xh} + \beta_{2h}(x))\right]. \tag{1.11}$$

To the first degree of approximation, biases and mean squared errors of $\hat{\bar{Y}}_{st1}, \hat{\bar{Y}}_{st2}, \hat{\bar{Y}}_{st3}$ and $\hat{\bar{Y}}_{st4}$ are given by

$$Bias(\hat{\bar{Y}}_{st1}) = (1/X_{SD})\sum_{h=1}^{k} W_h^2\gamma_h(R_{SD}S_{xh}^2 - S_{yxh}), \tag{1.12}$$

$$Bias(\hat{\bar{Y}}_{st2}) = (1/X_{SE})\sum_{h=1}^{k} W_h^2\gamma_h(R_{SE}S_{xh}^2 - S_{yxh}), \tag{1.13}$$

$$Bias(\hat{\bar{Y}}_{st3}) = (1/X_{US1})\sum_{h=1}^{k} W_h^2\gamma_h\beta_{2h}(x)(R_{US1}\beta_{2h}(x)S_{xh}^2 - S_{yxh}), \tag{1.14}$$

$$Bias(\hat{\bar{Y}}_{st4}) = (1/X_{US2})\sum_{h=1}^{k} W_h^2\gamma_h C_{xh}(R_{US2}C_{xh}S_{xh}^2 - S_{yxh}), \tag{1.15}$$

$$MSE(\hat{\bar{Y}}_{st1}) = \sum_{h=1}^{k} W_h^2\gamma_h(S_{yh}^2 + R_{SD}^2S_{xh}^2 - 2R_{SD}S_{yxh}), \tag{1.16}$$

$$MSE(\hat{\bar{Y}}_{st2}) = \sum_{h=1}^{k} W_h^2\gamma_h(S_{yh}^2 + R_{SK}^2S_{xh}^2 - 2R_{SK}S_{yxh}), \tag{1.17}$$

$$MSE(\hat{\bar{Y}}_{st3}) = \sum_{h=1}^{k} W_h^2\gamma_h(S_{yh}^2 + R_{US1}^2\beta_{2h}^2(x)S_{xh}^2 - 2R_{US1}\beta_{2h}(x)S_{yxh}), \tag{1.18}$$

$$MSE(\hat{\bar{Y}}_{st4}) = \sum_{h=1}^{k} W_h^2\gamma_h(S_{yh}^2 + R_{US2}^2C_{xh}^2S_{xh}^2 - 2R_{US2}C_{xh}S_{yxh}). \tag{1.19}$$

where $X_{SD} = \sum_{h=1}^{k} W_h(\bar{X}_h + C_{xh})$, $X_{SK} = \sum_{h=1}^{k} W_h(\bar{X}_h + \beta_{2h}(x))$,

$X_{US1} = \sum_{h=1}^{k} W_h(\bar{X}_h\beta_{2h}(x) + C_{xh})$, $\quad X_{US2} = \sum_{h=1}^{k} W_h(\bar{X}_h C_{xh} + \beta_{2h}(x))$,

$R_{SD} = (\sum_{h=1}^{k} W_h\,\bar{Y}_h) / \sum_{h=1}^{k} W_h\,(\bar{X}_h + C_{xh})$, $R_{SE} = (\sum_{h=1}^{k} W_h\,\bar{Y}_h)/(\sum_{h=1}^{k} W_h\,(\bar{X}_h + \beta_{2h}(x))$,

$R_{US1} = (\sum_{h=1}^{k} W_h\,\bar{Y}_h) / \sum_{h=1}^{k} W_h\,(\bar{X}_h\beta_{2h}(x) + C_{xh})$ and $R_{US2} = (\sum_{h=1}^{k} W_h\,\bar{Y}_h) / \sum_{h=1}^{k} W_h\,(\bar{X}_h C_{xh} + \beta_{2h}(x))$.

2. Proposed Ratio Estimator

Kadilar and Cingi (2006) suggested two ratio type estimators using information on coefficient of kurtosis $\beta_2(x)$ and correlation coefficient (ρ) between study variable and auxiliary variable in simple random sampling as

$$\hat{\bar{Y}}_{KS1} = \bar{y}\left[(\bar{X}\beta_2(x)+\rho)/(\bar{x}\beta_2(x)+\rho)\right]. \qquad (2.\ 1)$$

Here we propose $\hat{\bar{Y}}_{KS1}$ in stratified random sampling as

$$\hat{\bar{Y}}_{M1} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h\,(\bar{X}_h\beta_{2h}(x)+\rho_h)/\sum_{h=1}^{k} W_h\,(\bar{x}_h\beta_{2h}(x)+\rho_h)\right], \qquad (2.\ 2)$$

To obtain the bias and mean squared error expressions of the proposed estimator, we assume $\bar{y}_h = \bar{Y}_h(1+e_{0h})$ and $\bar{x}_h = \bar{X}_h(1+e_{1h})$ such that

$E(e_{0h}) = E(e_{1h}) = 0$, $E(e_{0h}^2) = \gamma_h C_{yh}^2$, $E(e_{1h}^2) = \gamma_h C_{xh}^2$ and $E(e_{0h}e_{1h}) = \gamma_h \rho_{yxh} C_{yh} C_{xh}$.

In terms of e_i's, $\hat{\bar{Y}}_{M1}$ can be written as

$$\hat{\bar{Y}}_{M1} = \sum_{h=1}^{k} W_h \bar{Y}_h (1+e_{0h}) \left\{ \frac{\sum_{h=1}^{k} W_h\left(\bar{X}_h\beta_{2h}(x)+\rho_h\right)}{\sum_{h=1}^{k} W_h\left\{\bar{X}_h(1+e_{1h})\beta_{2h}(x)+\rho_h\right\}} \right\},$$

$$\hat{\bar{Y}}_{M1} = \bar{Y}(1+e_0)(1+e_1)^{-1}.$$

where $e_0 = \dfrac{\sum_{h=1}^{k} W_h \bar{Y}_h e_{0h}}{\bar{Y}}$ and $e_1 = \dfrac{\sum_{h=1}^{k} W_h \bar{X}_h \beta_{2h}(x) e_{1h}}{\sum_{h=1}^{k} W_h\left(\bar{X}_h\beta_{2h}(x)+\rho_h\right)}$.

Now

$$E(e_0^2) = E\left(\frac{\sum_{h=1}^{k} W_h \bar{Y}_h e_{0h}}{\bar{Y}}\right)^2 = \frac{1}{\bar{Y}^2}\sum_{h=1}^{k} W_h^2 \gamma_h S_{yh}^2,$$

$$E(e_1^2) = E\left(\frac{\sum_{h=1}^{k} W_h \bar{X}_h \beta_{2h}(x) e_{1h}}{\sum_{h=1}^{k} W_h\left(\bar{X}_h\beta_{2h}(x)+\rho_h\right)}\right)^2 = \frac{1}{X_{M1}^2}\sum_{h=1}^{k} W_h^2 \gamma_h \beta_{2h}^2(x) S_{xh}^2 \text{ and}$$

$$E(e_0e_1)=E\left[\frac{\left(\sum_{h=1}^{k}W_h\bar{Y}_he_{0h}\right)\left(\sum_{h=1}^{k}W_h\bar{X}_h\beta_{2h}(x)e_{1h}\right)}{\bar{Y}\left(\sum_{h=1}^{k}W_h\bar{X}_h\left(\beta_{2h}(x)+\rho_h\right)\right)}\right] = \frac{1}{\bar{Y}X_{M1}}\sum_{h=1}^{L}W_h^2\gamma_h\beta_{2h}(x)S_{yxh} .$$

To the first degree of approximation the bias and mean squared error of $\hat{\bar{Y}}_{M1}$ are obtained as

$$Bias(\hat{\bar{Y}}_{M1})=(1/X_{M1})\sum_{h=1}^{k}W_h^2\gamma_h\beta_{2h}(x)(R_{M1}\beta_{2h}(x)S_{xh}^2-S_{yxh}), \qquad (2.3)$$

$$MSE(\hat{\bar{Y}}_{M1})=\sum_{h=1}^{k}W_h^2\gamma_h(S_{yh}^2+R_{M1}^2\beta_{2h}^2(x)S_{xh}^2-2R_{M1}\beta_{2h}(x)S_{yxh}), \qquad (2.4)$$

where $X_{M1}=\sum_{h=1}^{k}W_h(\bar{X}_h\beta_{2h}(x)+\rho_h)$ and $R_{M1}=\dfrac{\sum_{h=1}^{k}W_h\bar{Y}_h}{\sum_{h=1}^{k}\left(W_h\bar{X}_h\beta_{2h}(x)+\rho_h\right)}$.

Expressions 2.3 and 2.4 contains population parameters such as population mean, population mean squares, coefficients of kurtosis and covariances for each stratum. Hence expressions for bias and mean squared error of the suggested estimator $\hat{\bar{Y}}_{M1}$ in (2.3) and (2.4) can not be used in real situation so estimator for the bias and mean squared error of the Bias and MES of the suggested estimator are given below:

$$Bias(\hat{\bar{Y}}_{M1})=(1/X_{M1})\sum_{h=1}^{k}W_h^2\gamma_h\beta_{2h}(x)(R_{M1}\beta_{2h}(x)S_{xh}^2-S_{yxh}) \qquad (2.5)$$

$$MSE(\hat{\bar{Y}}_{M1})=\sum_{h=1}^{k}W_h^2\gamma_h(S_{yh}^2+R_{M1}^2\beta_{2h}^2(x)S_{xh}^2-2R_{M1}\beta_{2h}(x)S_{yxh}) \qquad (2.6)$$

3. Efficiency Comparisons

In stratified random sampling variance of usual unbiased estimator $\bar{y}_{st}$ is given by

$$V(\bar{y}_{st})=\sum_{h=1}^{L}W_h^2\gamma_hS_{yh}^2 \qquad (3.1)$$

From (1.3), (1.16), (1.17), (1.18), (1.19), (2.4) and (3.1), it follows that

(i) $MSE(\hat{\bar{Y}}_{M1})<V(\bar{y}_{st})$ if

$$R_{M1}<\frac{2C}{D}, \qquad (3.2)$$

(ii) $MSE(\hat{\bar{Y}}_{M1})<MSE(\hat{\bar{Y}}_{RC})$ if

$$R_{M1}\left(R_{M1}D-2C\right)-R\left(RB-2A\right)<0\,, \tag{3.3}$$

(iii) $MSE(\hat{\bar{Y}}_{M1})<MSE(\hat{\bar{Y}}_{st1})$ if

$$R_{M1}\left(R_{M1}D-2C\right)-R_{SD}\left(R_{SD}B-2A\right)<0\,, \tag{3.4}$$

(iv) $MSE(\hat{\bar{Y}}_{M1})<MSE(\hat{\bar{Y}}_{st2})$ if

$$R_{M1}\left(R_{M1}D-2C\right)-R_{SK}\left(R_{SK}B-2A\right)<0\,, \tag{3.5}$$

(v) $MSE(\hat{\bar{Y}}_{M1})<MSE(\hat{\bar{Y}}_{st3})$ if

$$R_{M1}\left(R_{M1}D-2C\right)-R_{US1}\left(R_{US1}D-2C\right)<0\,, \tag{3.6}$$

(vi) $MSE(\hat{\bar{Y}}_{M1})<MSE(\hat{\bar{Y}}_{st4})$ if

$$R_{M1}\left(R_{M1}D-2C\right)-R_{US2}\left(R_{US2}F-2E\right)<0\,, \tag{3.7}$$

where $A=\sum_{h=1}^{k}W_h^2\gamma_h S_{yxh}$, $\quad B=\sum_{h=1}^{k}W_h^2\gamma_h S_{xh}^2$,

$C=\sum_{h=1}^{k}W_h^2\gamma_h\beta_{2h}(x)S_{yxh}$, $\quad D=\sum_{h=1}^{k}W_h^2\gamma_h\beta_{2h}^2(x)S_{xh}^2$,

$E=\sum_{h=1}^{k}W_h^2\gamma_h C_{xh}S_{yxh}$ and $F=\sum_{h=1}^{k}W_h^2\gamma_h C_{xh}^2 S_{xh}^2$.

4. Bias Comparisons

Comparing (1.2) with (2.3), it is observed that the bias of proposed estimator $\hat{\bar{Y}}_{M1}$ would be less than the bias of combined ratio estimator $\hat{\bar{Y}}_{RC}$ i.e. $\left|B(\hat{\bar{Y}}_{M1})\right|<\left|B(\hat{\bar{Y}}_{RC})\right|$ if

$$\left(R_{M1}D-C\right)^2<\frac{X_{M1}^2}{\bar{X}^2}\left(RB-A\right)^2. \tag{4.1}$$

From (1.12) and (2.3)it follows that the bias of proposed estimator $\hat{\bar{Y}}_{M1}$ would be less than the bias of estimator $\hat{\bar{Y}}_{st1}$ i.e. $\left|B(\hat{\bar{Y}}_{M1})\right|<\left|B(\hat{\bar{Y}}_{st1})\right|$ if

$$\left(R_{M1}D-C\right)^2<\frac{X_{M1}^2}{X_{SD}^2}\left(R_{SD}B-A\right)^2. \tag{4.2}$$

Comparison of (1.13) and (2.3)shows that the bias of suggested estimator would be less than the bias of $\hat{\bar{Y}}_{st2}$ i.e. $\left|B(\hat{\bar{Y}}_{M1})\right|<\left|B(\hat{\bar{Y}}_{st2})\right|$ if

$$\left(R_{M1}D - C\right)^2 < \frac{X_{M1}^2}{X_{SK}^2}\left(R_{SK}B - A\right)^2. \qquad (4.3)$$

From (1.14) and (2.3), it can be concluded that the suggested estimator has less bias than the bias of Kadilar and Cingi (2003) estimator $\hat{\bar{Y}}_{st3}$ i.e. $\left|B(\hat{\bar{Y}}_{M1})\right| < \left|B(\hat{\bar{Y}}_{st3})\right|$ if

$$\left(R_{M1}D - C\right)^2 < \frac{X_{M1}^2}{X_{US1}^2}\left(R_{US1}D - C\right)^2. \qquad (4.4)$$

From (1.15) and (2.3), it can be seen that proposed estimator $\hat{\bar{Y}}_{M1}$ has less bias in comparison to the bias of Kadilar and Cingi (2003) estimator $\hat{\bar{Y}}_{st4}$ i.e. $\left|B(\hat{\bar{Y}}_{M1})\right| < \left|B(\hat{\bar{Y}}_{st4})\right|$ if

$$\left(R_{M1}D - C\right)^2 < \frac{X_{M1}^2}{X_{US2}^2}\left(R_{US3}F - E\right)^2. \qquad (4.5)$$

5. Generalized Version of Proposed Estimator

Using power transformation in (2.1) the generalized version of $\hat{\bar{Y}}_{M1}$ is defined as

$$\hat{\bar{Y}}_{M1\alpha} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h(\bar{X}_h\beta_{2h}(x) + \rho_h) / \sum_{h=1}^{k} W_h(\bar{x}_h\beta_{2h}(x) + \rho_h)\right]^{\alpha 1}, \qquad (5.1)$$

where α_1 is a suitably chosen scalar.

The bias and mean squared error of $\hat{\bar{Y}}_{M1\alpha}$ to the first degree of approximation are obtained as

$$Bias(\hat{\bar{Y}}_{M1\alpha}) = \frac{\alpha_1}{X_{M1}} E\left[\sum_{h=1}^{k} W_h^2\gamma_h\beta_{2h}(x)\left\{\frac{(1+\alpha_1)}{2}\beta_{2h}(x)R_{M1}S_{xh}^2 - S_{yxh}\right\}\right] \qquad (5.2)$$

and

$$MSE(\hat{\bar{Y}}_{M1\alpha}) = \left[\sum_{h=1}^{k} W_h^2\gamma_h(S_{yh}^2 + \alpha_1^2 R_{M1}^2\beta_{2h}^2(x)S_{xh}^2 - 2\alpha_1 R_{M1}\beta_{2h}(x)S_{yxh})\right]. \qquad (5.3)$$

The value of α_1, for which mean squared error of $\hat{\bar{Y}}_{M1\alpha}$ would be minimum, can be obtained using principle of maxima and minima

i.e. $$\frac{\partial MSE(\hat{\bar{Y}}_{M1\alpha})}{\partial \alpha_1} = 0$$

which provides

$$\alpha_1 = [\sum_{h=1}^{k} W_h^2 \gamma_h \beta_{2h}(x) S_{yxh} / R_{M1} \sum_{h=1}^{k} W_h^2 \gamma_h \beta_{2h}^2(x) S_{xh}^2] \quad . \tag{5.4}$$

This is the value of α_1 for which we get minimum value of $MSE\left(\hat{\bar{Y}}_{M1\alpha}\right)$.

Substituting the value of α_1 in (5.3), we get minimum mean squared error of $\hat{\bar{Y}}_{M1\alpha}$ as

$$\min . MSE(\hat{\bar{Y}}_{M1\alpha}) = \left[\sum_{h=1}^{k} W_h^2 \gamma_h S_{yh}^2 - (\sum_{h=1}^{k} W_h^2 \gamma_h \beta_{2h}(x) S_{yxh})^2 / (\sum_{h=1}^{k} W_h^2 \gamma_h \beta_{2h}^2(x) S_{xh}^2) \right] . \tag{5.5}$$

Efficiency Comparisons for Generalized Version of Proposed Estimator $\hat{\bar{Y}}_{M1}$

Comparing (3.1)and (5.3), it is observed that the proposed estimator $\hat{\bar{Y}}_{M1\alpha}$ would be more efficient than usual unbiased estimator $\bar{y}_{st}$ if

$$\left.\begin{array}{ll} \text{either} & 0 < \alpha_1 < \dfrac{2C}{DR_{M1}} \\ \text{or} & \dfrac{2C}{DR_{M1}} < \alpha_1 < 0 \end{array}\right\} . \tag{5.6}$$

Condition for which the proposed estimator $\hat{\bar{Y}}_{M1\alpha}$ would be more efficient than combined ratio estimator $\hat{\bar{Y}}_{RC}$ can be obtained by comparing (1.3) and (5.3) as

$$\begin{aligned} \frac{C}{DR_{M1}} - \sqrt{\left(\frac{C}{DR_{M1}}\right)^2 + \frac{R}{R_{M1}^2 D}\left(RB - 2A\right)} < \alpha \\ < \frac{C}{DR_{M1}} + \sqrt{\left(\frac{C}{DR_{M1}}\right)^2 + \frac{R}{R_{M1}^2 D}\left(RB - 2A\right)} \end{aligned} . \tag{5.7}$$

From (1.16) and (5.3), it can be seen that the proposed estimator $\hat{\bar{Y}}_{M1\alpha}$ is more efficient than estimator $\hat{\bar{Y}}_{st1}$, if

$$\begin{aligned} \frac{C}{DR_{M1}} - \sqrt{\left(\frac{C}{DR_{M1}}\right)^2 + \frac{R_{SD}}{R_{M1}^2 D}\left(R_{SD} B - 2A\right)} < \alpha \\ < \frac{C}{DR_{M1}} + \sqrt{\left(\frac{C}{DR_{M1}}\right)^2 + \frac{R_{SD}}{R_{M1}^2 D}\left(R_{SD} B - 2A\right)} \end{aligned} . \tag{5.8}$$

Comparison of (1.17) and (5.3) shows that the proposed estimator $\hat{\bar{Y}}_{M1\alpha}$ is more efficient than estimator $\hat{\bar{Y}}_{st2}$, if

$$\frac{C}{DR_{M1}} - \sqrt{\left(\frac{C}{DR_{M1}}\right)^2 + \frac{R_{SK}}{R_{M1}^2 D}\left(R_{SK}B - 2A\right)} < \alpha < \frac{C}{DR_{M1}} + \sqrt{\left(\frac{C}{DR_{M1}}\right)^2 + \frac{R_{SK}}{R_{M1}^2 D}\left(R_{SK}B - 2A\right)} \quad . \qquad (5.9)$$

From (1.18) and (5.3), it can be seen that the proposed estimator $\hat{\bar{Y}}_{M1\alpha}$ is more efficient than estimator $\hat{\bar{Y}}_{st3}$, if

$$\frac{C}{DR_{M1}} - \sqrt{\left(\frac{C}{DR_{M1}}\right)^2 + \frac{R_{US1}}{R_{M1}^2 D}\left(R_{US1}D - 2C\right)} < \alpha < \frac{C}{DR_{M1}} + \sqrt{\left(\frac{C}{DR_{M1}}\right)^2 + \frac{R_{US1}}{R_{M1}^2 D}\left(R_{US1}D - 2C\right)} \quad . \qquad (5.10)$$

From (1.19) and (5.3) it is observed that proposed estimator $\hat{\bar{Y}}_{M1\alpha}$ would be more efficient than estimator $\hat{\bar{Y}}_{st4}$, if

$$\frac{C}{DR_{M1}} - \sqrt{\left(\frac{C}{DR_{M1}}\right)^2 + \frac{R_{US2}}{R_{M1}^2 D}\left(R_{US2}F - 2E\right)} < \alpha < \frac{C}{DR_{M1}} + \sqrt{\left(\frac{C}{DR_{M1}}\right)^2 + \frac{R_{US2}}{R_{M1}^2 D}\left(R_{US2}F - 2E\right)} \quad . \qquad (5.11)$$

From (2.4) and (5.3), it can be seen that the proposed estimator $\hat{\bar{Y}}_{M1\alpha}$ is more efficient than estimator $\hat{\bar{Y}}_{M1}$, if

$$\left.\begin{array}{ll} \text{either} & 1 < \alpha < \dfrac{2C}{DR_{M1}} - 1 \\ \text{or} & \dfrac{2C}{DR_{M1}} - 1 < \alpha < 1 \end{array}\right\} . \qquad (5.12)$$

6. Empirical Study

To compare the proposed estimator numerically with other estimators, we are considering two real populations. Descriptions of the populations are given below:

Population I [Source: Singh and Mangat (1996)]

Y: Weight of juce in grams

X : Weight of sugar canes in grams

$\bar{X}_1 = 366.6667$	$\bar{X}_2 = 310.8333$	$\bar{X}_3 = 317.1429$
$\bar{Y}_1 = 135$	$\bar{Y}_2 = 99.1667$	$\bar{Y}_3 = 80.7142$
$\beta_{x1} = 2.2865$	$\beta_{x2} = 3.2689$	$\beta_{x3} = 3.1306$
$C_{x1} = 0.1419$	$C_{x2} = 0.1395$	$C_{x3} = 0.1695$
$C_{y1} = 0.0662$	$C_{y2} = 0.1518$	$C_{y3} = 0.1358$
$S_{x1} = 52.0256$	$S_{x2} = 43.3712$	$S_{x3} = 53.7631$
$S_{y1} = 8.9443$	$S_{y2} = 15.0504$	$S_{y3} = 10.9653$
$\rho_1 = 0.9456$	$\rho_2 = 0.9482$	$\rho_3 = 0.7532$
$\gamma_1 = 0.1267$	$\gamma_2 = 0.0433$	$\gamma_3 = 0.1028$
$\omega_1^2 = 0.0576$	$\omega_2^2 = 0.2304$	$\omega_3^2 = 0.0784$

Population II [Source: Kadilar and Cingi (2003)]

Y: Apple production as a study variable

X: No. of Apple trees

In this population data were collected from 854 villages of Turkey in 1999 (Source: Institute of Statistics, Republic of Turkey).

$\bar{X}_1 = 24375$	$\bar{X}_2 = 27421$	$\bar{X}_3 = 72409$	$\bar{X}_4 = 74365$	$\bar{X}_5 = 26441$	$\bar{X}_6 = 9844$
$\bar{Y}_1 = 1536$	$\bar{Y}_2 = 2212$	$\bar{Y}_3 = 9384$	$\bar{Y}_4 = 5588$	$\bar{Y}_5 = 967$	$\bar{Y}_6 = 404$
$\beta_{x1} = 25.71$	$\beta_{x2} = 34.57$	$\beta_{x3} = 26.14$	$\beta_{x4} = 97.60$	$\beta_{x5} = 27.47$	$\beta_{x6} = 28.10$
$C_{x1} = 2.02$	$C_{x2} = 2.10$	$C_{x3} = 2.22$	$C_{x4} = 3.84$	$C_{x5} = 1.72$	$C_{x6} = 1.91$
$C_{y1} = 4.18$	$C_{y2} = 5.22$	$C_{y3} = 3.19$	$C_{y4} = 5.13$	$C_{y5} = 2.47$	$C_{y6} = 2.34$
$S_{x1} = 49189$	$S_{x2} = 57461$	$S_{x3} = 160757$	$S_{x4} = 285603$	$S_{x5} = 45403$	$S_{x6} = 18794$
$S_{y1} = 6425$	$S_{y2} = 11552$	$S_{y3} = 29907$	$S_{y4} = 28643$	$S_{y5} = 2390$	$S_{y6} = 946$
$\rho_1 = 0.82$	$\rho_2 = 0.86$	$\rho_3 = 0.90$	$\rho_4 = 0.99$	$\rho_5 = 0.71$	$\rho_6 = 0.89$
$\gamma_1 = 0.102$	$\gamma_2 = 0.049$	$\gamma_3 = 0.016$	$\gamma_4 = 0.009$	$\gamma_5 = 0.138$	$\gamma_6 = 0.006$
$\omega_1^2 = 0.015$	$\omega_2^2 = 0.015$	$\omega_3^2 = 0.012$	$\omega_4^2 = 0.04$	$\omega_5^2 = 0.057$	$\omega_6^2 = 0.041$

Table 6.1: Percent Relative Efficiencies of $\bar{y}_{st}$, $\hat{\bar{Y}}_{RC}$, $\hat{\bar{Y}}_{st1}$, $\hat{\bar{Y}}_{st2}$, $\hat{\bar{Y}}_{st3}$, $\hat{\bar{Y}}_{st4}$, $\hat{\bar{Y}}_{M1}$ and $\hat{\bar{Y}}_{M1\alpha}$ with respect to $\bar{y}_{st}$

Estimators	**Percent Relative Efficiency (•, $\bar{y}_{st}$,)**	
	Population I	**Population II**
$\bar{y}_{st}$	100.00	100.00
$\hat{\bar{Y}}_{RC}$	214.84	317.65
$\hat{\bar{Y}}_{st1}$	215.07	317.55
$\hat{\bar{Y}}_{st2}$	219.38	317.37
$\hat{\bar{Y}}_{st3}$	240.73	2356
$\hat{\bar{Y}}_{st4}$	206.85	326.38
$\hat{\bar{Y}}_{M1}$	241.17	2356
$\hat{\bar{Y}}_{M1\alpha(\alpha_{1(opt)})}$	383.70	248.10

Table 6.2: Percent Relative Efficiencies of $\hat{\bar{Y}}_{M1\alpha}$ for different values of α_1 .

α_i	**Population I**	**Population II**
0.00	100.00	100.00
0.25	178.70	144.31
0.50	317.22	201.76
0.75	374.64	245.12
1.00	241.17	2356
1.25	131.85	178.03
$\alpha_{iOpt.}$	383.70 $\alpha_{1(opt)} = 0.6866$	248.10 $\alpha_{1(opt)} = 0.8248$

Table 6.3: Range of α_1 in which generalized estimator $\hat{\hat{Y}}_{M1\alpha}$ would be more efficient than $\bar{y}_{st}$, $\hat{\hat{Y}}_{RC}$, $\hat{\hat{Y}}_{st1}$, $\hat{\hat{Y}}_{st2}$, $\hat{\hat{Y}}_{st3}$, $\hat{\hat{Y}}_{st4}$ and $\hat{\hat{Y}}_{M1}$.

Estimator	**POPULATION I**	**POPULATION II**
$\bar{y}_{st}$	(0.00, 1.37)	(0.00, 1.36)
$\hat{\hat{Y}}_{RC}$	(0.32, 1.05)	(0.50, 1.14)
$\hat{\hat{Y}}_{st1}$	(0.32, 1.04)	(0.50, 1.14)
$\hat{\hat{Y}}_{st2}$	(0.34, 1.03)	(0.50, 1.14)
$\hat{\hat{Y}}_{st3}$	(-0.21, 1.58)	(0.01, 1.63)
$\hat{\hat{Y}}_{st4}$	(0.31, 1.06)	(0.49 1.15)
$\hat{\hat{Y}}_{M1}$	(0.37, 1.00)	(0.64, 1.00)

Table 6.1 exhibits that there is substantial gain in efficiency by using the suggested estimator $\hat{\hat{Y}}_{M1}$ over unbiased estimator $\bar{y}_{st}$, combined ratio estimator $\hat{\hat{Y}}_{RC}$, and estimators $\hat{\hat{Y}}_{st1}$, $\hat{\hat{Y}}_{st2}$, $\hat{Y}_{st3}$ and $\hat{\hat{Y}}_{st4}$ given by Kadilar and Cingi (2003). Table 6.2 demonstrates that the larger gain in efficiency in the vicinity of the optimum value of the scalar α_1 is observed and the maximum gain in efficiency at the optimum value of α_1 (say $\alpha_{(opt)}$).

Table 6.2 shows the range of α_1 in which proposed generalized estimator $\hat{\hat{Y}}_{M1\alpha}$ performs better than $\hat{\hat{Y}}_{RC}$, $\hat{\hat{Y}}_{st1}$, $\hat{\hat{Y}}_{st2}$, $\hat{Y}_{st3}$ and $\hat{\hat{Y}}_{st4}$.

At the end, we concl4'ude that there is enough scope of selecting the values of α_1 to obtain better estimators from the proposed estimators even when the scalar α_1 departs from its exact optimum value $\alpha_{1(opt)}$.

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A SEPARATE RATIO-CUM-PRODUCT ESTIMATOR OF POPULATION MEAN USING AUXILIARY INFORMATION IN STRATIFIED RANDOM SAMPLING

Rajesh Tailor[1], Med Ram Verma*[2], Sunil Chouhan[1],
Ritesh Tailor[3] and Priyanka Malviya[1]
[1]School of Studies in Statistics, Vikram University, Ujjain-456010 (M.P.)
[2]Division of LES &IT, ICAR-IVRI, Izatnagar, Bareilly, UP
[3]Institute of Wood Science and Technology (IWST) Bangalore, India
*Corresponding author E-mail: medramverma@rediffmail.com

Abstract:

This chapter deals with the estimation of population mean in stratified random sampling. A separate ratio cum product estimator using coefficient of kurtosis and coefficient of variation of auxiliary variable is proposed. The bias and mean squared error of the proposed estimator have been derived under large sample approximations. Theoretical conditions for which the proposed estimator is more efficient than other estimators considered are obtained. Empirical study is also carried out to support the theoretical results.

MSC: 94A20

Keywords: Population mean, Mean squared error, Bias, Coefficient of kurtosis, Coefficient of variance, Stratified random sampling.

1. Introduction

It is well known that if information on auxiliary varaiate (s) is suitably used then it may provide more efficient estimators. It is also established that stratified random sampling prove to be more efficient than simple random sampling in planning surveys. Many researchers including Singh (1967), Sisodia and Dwivedi (1981), Singh and Kakran (1993), Upadhyaya and Singh (1999) and Singh *et al.* (2004), Kadilar and Cingi (2003, 2005, 2006), Singh and Vishvakarma (2005, 2006) etc. made use of auxiliary information at estimation stage. In stratified random sampling ratio estimators can be defined in two ways: Combined ratio estimators and Separate ratio estimators. In cases when line of regression on y on x passes through origin within each stratum, the separate ratio estimator will be more precise than combined one. In this chapter a separate ratio- cum product estimator of population mean in stratified random sampling is suggested.

Consider a finite population $U = U_1, U_2, ..., U_N$ of size N and it is divided into L strata of

Size N_h $(h=1,2,......L)$. Let y and x be the study variate and auxiliary variate respectively taking values y_{hi} and x_{hi} $(h=1,2,....L;\ i=1,2,....,N_h)$. A sample of size n_h is drawn from each stratum which constitutes a sample of size $n=\sum_{h=1}^{L} n_h$

Notations:

y : Study variate, x : Auxiliary variate,

y_{hi} : Observation on i^{th} unit of h^{th} stratum on study variate,

x_{hi} : Observation on i^{th} unit of h^{th} stratum on auxiliary variate,

$\bar{y}_h$:Sample mean of study variate for h^{th} stratum,

$\bar{x}_h$:Sample mean of auxiliary variate for h^{th} stratum,

$\bar{y}=\frac{1}{n}\sum_{h=1}^{L}\sum_{i=1}^{n_h} y_{hi}$: Sample mean of study variate,

$\bar{x}=\frac{1}{n}\sum_{h=1}^{L}\sum_{i=1}^{n_h} x_{hi}$: Sample mean of auxiliary variate,

$\bar{Y}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi}$: Population mean of study variate,

$\bar{X}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi}$: Population mean of auxiliary variate,

$\bar{y}_{st}=\frac{1}{n}\sum_{h=1}^{L} W_h\bar{y}_h$: Unbiased estimator of population mean $\bar{Y}$ in stratified random sampling,

$\bar{x}_{st}=\frac{1}{n}\sum_{h=1}^{L} W_h\bar{x}_h$: Unbiased estimator of population mean $\bar{X}$ in stratified random sampling,

$W_h=\frac{N_h}{N}$: Stratum weight of h^{th} stratum.

The Combined ratio estimator to estimate population mean $\bar{Y}$ is given by

$$\bar{y}_{RC}=\bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right) \tag{1.1}$$

Here $\bar{X}$ is assumed to be known.

The classical separate ratio estimator for population mean $\bar{Y}$ is defined as

$$\bar{y}_{RS}=\sum_{h=1}^{L} W_h\bar{y}_h\,\frac{\bar{X}_h}{\bar{x}_h} \tag{1.2}$$

Bias and mean squared error of Separate ratio estimator $\bar{y}_{RS}$ are

$$B(\bar{y}_{RS}) = \sum_{h=1}^{L} W_h \ \gamma_h \bar{Y}_h \left[C_{xh}^2 - \rho_{yxh} C_{yh} C_{xh}\right] \tag{1.3}$$

$$MSE(\bar{y}_{RS}) = \sum_{h=1}^{L} W_h^{\ 2} \gamma_h \bar{Y}_h^{\ 2} \left[S_{yh}^2 + R_h^2 S_{xh}^2 + 2R_h \rho_{yxh} S_{yh} S_{xh}\right] \tag{1.4}$$

Upadhyaya and Singh (1999) used coefficient of variation (C_x) and coefficient of kurtosis $(\beta_2(x))$ of auxiliary variate *x* and proposed a ratio type estimator as

$$t_1 = \bar{y}\left(\frac{\bar{X}\beta_2(x) + C_x}{\bar{x}\beta_2(x) + C_x}\right) \tag{1.5}$$

Kadilar and Cingi (2003) defined t_1 in stratified random sampling as

$$t_{1st} = \bar{y}_{st}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h \beta_{2h}(x) + C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h \beta_{2h}(x) + C_{xh}\right)}\right) \tag{1.6}$$

2. Proposed estimator:

We propose a Separate ratio estimator using coefficient of variation (C_x) and coefficient of kurtosis $(\beta_2(x))$ of auxiliary variate *x* based on Upadhyaya and Singh (1999) as

$$t_{\alpha s} = \sum_{h=1}^{L} W_h \bar{y}_h \left[\alpha_h \left\{\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h \beta_{2h}(x) + C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h \beta_{2h}(x) + C_{xh}\right)}\right\} + (1 - \alpha_h)\left\{\frac{\sum_{h=1}^{L} W_h\left(\bar{x}_h \beta_{2h}(x) + C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h \beta_{2h}(x) + C_{xh}\right)}\right\}\right]$$

Where α_h are suitably chosen scalars.

To obtain bias and mean squared error of proposed estimator $t_{\alpha s}$, we write

$\bar{y}_h = \bar{Y}_h(1 + e_{0h})$, $\bar{x}_h = \bar{X}_h(1 + e_{1h})$ such that

$E(e_{0h}) = E(e_{1h}) = 0$, $E(e_{0h}^2) = \gamma_h C_{yh}^2$, $E(e_{1h}^2) = \gamma_h C_{xh}^2$

and $E(e_{0h} e_{1h}) = \gamma_h \rho_{yxh} C_{yh} C_{xh} = \gamma_h S_{yxh}$

$$t_{\alpha s} = \sum_{h=1}^{L} W_h \bar{y}_h \left[\alpha_h \left\{\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h \beta_{2h}(x) + C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h \beta_{2h}(x) + C_{xh}\right)}\right\} + (1 - \alpha_h)\left\{\frac{\sum_{h=1}^{L} W_h\left(\bar{x}_h \beta_{2h}(x) + C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h \beta_{2h}(x) + C_{xh}\right)}\right\}\right]$$

$$= \sum_{h=1}^{L} W_h \bar{Y}_h (1+e_{0h}) \left[\alpha_h \{1+\lambda_h e_{1h}\}^{-1} + (1-\alpha_h)\{1+\lambda_h e_{1h}\} \right]$$

where

$$\lambda_h = \frac{\bar{X}_h \beta_{2h}(x)}{\bar{X}_h \beta_{2h}(x) + C_{xh}}$$

To the first degree of approximations bias and mean squared error of proposed estimator t_α are obtained as

$$B(t_{\alpha s}) = \sum_{h=1}^{L} W_h \bar{Y}_h \left[\lambda_h^2 C_{xh}^2 + \lambda_h (1-2\alpha_h) \rho_{yxh} C_{yh} C_{xh} \right] \quad (2.1)$$

$$MSE(t_{\alpha s}) = \sum_{h=1}^{L} W_h \gamma_h \bar{Y}_h^{\,2} \left[S_{yh}^2 + \lambda_h^2 (1-2\alpha_h)^2 R_h^2 S_{xh}^2 + 2\lambda_h (1-2\alpha_h) R_h \rho_{yxh} S_{yh} S_{xh} \right] \quad (2.2)$$

where $R_h = \dfrac{\bar{Y}_h}{\bar{X}_h}$

3. Efficiency Comparisons:

Variance of usual unbiased estimator of mean $\bar{y}_{st}$ in stratified random sampling is

$$V(\bar{y}_{st}) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2 \quad (3.1)$$

Comparison of (2.2) and (3.1) shows that proposed estimator $t_{\alpha s}$ would be more efficient than unbiased estimator $\bar{y}_{st}$ i.e. $MSE(t_{\alpha s}) < \mathrm{Var}(\bar{y}_{st})$ If

$$\left.\begin{array}{ll} \text{Either} & \dfrac{1}{2} < \alpha_h < \dfrac{1}{2}\left(1+\dfrac{2K_h}{\lambda_h}\right) \\ \text{or} & \dfrac{1}{2}\left(1+\dfrac{2K_h}{\lambda_h}\right) < \alpha_h < \dfrac{1}{2} \end{array}\right\} \quad (3.2)$$

where $K_h = \rho_{yxh} \dfrac{S_{yh}}{S_{xh}}$

Comparing (1.4) and (2.2) it is observed that proposed estimator $t_{\alpha s}$ would be more efficient than classical separate ratio estimator $\bar{y}_{RS}$

i.e. $MSE(t_{\alpha s}) < MSE(\bar{y}_{RS})$ if

$$\left.\begin{array}{ll} \text{either} & \frac{1}{2}\left(1-\frac{1}{\lambda_h}\right) < \alpha_h < \frac{1}{2}\left\{1+\lambda_h\left(1+\frac{2K_h}{R_h}\right)\right\} \\ \text{or} & \frac{1}{2}\left\{1+\lambda_h\left(1+\frac{2K_h}{R_h}\right)\right\} < \alpha_h < \frac{1}{2}\left(1-\frac{1}{\lambda_h}\right)\frac{1}{2} \end{array}\right\} \tag{3.3}$$

4. Estimator at optimum α_h

Value of α_h for which proposed estimator gives optimum results i.e. minimum mean squared error, can be obtained using principle of maxima and minima

i.e. $\frac{\partial MSE(t_{\alpha S})}{\partial \alpha_h} = 0$ which provides

$$\alpha_h = \frac{1}{2}\left[1+\frac{\rho_{yxh}S_{yh}}{R_h\lambda_h S_{xh}}\right] = \alpha_{hopt}\,(\text{say}) \tag{4.1}$$

This is the value of α for which $MSE(t_\alpha)$ gives minimum mean squared error of t_α .

Substituting the value of α in proposed estimator (3.1.2.1) the optimum estimator $t_{\alpha(opt)}$ can be expressed as

$$t_{\alpha sopt} = \sum_{h=1}^{L} W_h \bar{y}_h \left[\frac{1}{2}\left(1+\frac{\rho_{yxh}S_{yh}}{R_h\lambda_h S_{xh}}\right)\left\{\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h\beta_{2h}(x)+C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h\beta_{2h}(x)+C_{xh}\right)}\right\} + \left(\frac{1}{2}-\frac{\rho_{yxh}S_{yh}}{2R_h\lambda_h S_{xh}}\right)\left\{\frac{\sum_{h=1}^{L} W_h\left(\bar{x}_h\beta_{2h}(x)+C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h\beta_{2h}(x)+C_{xh}\right)}\right\}\right]$$

and minimum mean squared error of $t_{\alpha S}$ is

$$MSE(t_{\alpha Sopt}) = \sum_{h=1}^{L} W_h \gamma_h{}^2 S_{yh}\left(1-\rho_{yxh}^2\right)$$

Remark 1. For $\alpha_h = 1-f_h = 1-\frac{n_h}{N_h}$, proposed estimator $t_{\alpha S}$ turns to

$$t_{S1} = \sum_{h=1}^{L} W_h \bar{y}_h \left[(1-f_h)\left\{\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h\beta_{2h}(x)+C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h\beta_{2h}(x)+C_{xh}\right)}\right\} + f_h\left\{\frac{\sum_{h=1}^{L} W_h\left(\bar{x}_h\beta_{2h}(x)+C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h\beta_{2h}(x)+C_{xh}\right)}\right\}\right]$$

Bias and MSE of t_{S2} are obtained by substituting $\alpha_h = 1-f_h$ as

$$B(t_{S1}) = \sum_{h=1}^{L} W_h \bar{Y}_h \left[\lambda_h^2 C_{xh}^2 + \lambda_h (2f_h - 1)\rho_{yxh} C_{yh} C_{xh}\right] \quad (4.2)$$

$$MSE(t_{S1}) = \sum_{h=1}^{L} W_h \gamma_h \bar{Y}_h^2 \left[S_{yh}^2 + \lambda_h^2 (2f_h - 1)^2 R_h^2 S_{xh}^2 + 2\lambda_h (2f_h - 1) R_h \rho_{yxh} S_{yh} S_{xh}\right] \quad (4.3)$$

Remark 2. For $\alpha_h = 1 + f_h = 1 + \frac{n_h}{N_h}$, proposed estimator $t_{\alpha S}$ turns to

$$t_{S2} = \sum_{h=1}^{L} W_h \bar{y}_h \left[(1 + f_h) \left\{ \frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h \beta_{2h}(x) + C_{xh}\right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_h \beta_{2h}(x) + C_{xh}\right)} \right\} - f_h \left\{ \frac{\sum_{h=1}^{L} W_h \left(\bar{x}_h \beta_{2h}(x) + C_{xh}\right)}{\sum_{h=1}^{L} W_h \left(\bar{X}_h \beta_{2h}(x) + C_{xh}\right)} \right\} \right]$$

Bias and MSE of t_{S2} are obtained by substituting $\alpha_h = 1 + f_h$ as

$$B(t_{S2}) = \sum_{h=1}^{L} W_h \bar{Y}_h \left[\lambda_h^2 C_{xh}^2 - \lambda_h (1 + 2f_h)\rho_{yxh} C_{yh} C_{xh}\right] \quad (4.4)$$

$$MSE(t_{S2}) = \sum_{h=1}^{L} W_h \gamma_h \bar{Y}_h^2 \left[S_{yh}^2 + \lambda_h^2 (1 + 2f_h)^2 R_h^2 S_{xh}^2 - 2\lambda_h (1 + 2f_h) R_h \rho_{yxh} S_{yh} S_{xh}\right] \quad (4.5)$$

5. Empirical Study

The performance of the proposed estimator in comparison to other estimators is tested on five natural data sets.

For comparison of different estimators we are calculating percent relative efficiency (PRE)

$$PRE(\bar{y}_{RC}, \bar{y}_{st}) = \frac{V(\bar{y}_{st})}{MSE(\bar{y}_{RC})} \times 100$$

$$PRE(t_{RS}, \bar{y}_{st}) = \frac{V(\bar{y}_{st})}{MSE(t_{RS})} \times 100$$

$$PRE(t_{\alpha(opt)S}, \bar{y}_{st}) = \frac{V(\bar{y}_{st})}{MSE(t_{\alpha(opt)S})} \times 100$$

$$PRE(t_{S1}, \bar{y}_{st}) = \frac{V(\bar{y}_{st})}{MSE(t_{S1})} \times 100$$

$$PRE(t_{S2}, \bar{y}_{st}) = \frac{V(\bar{y}_{st})}{MSE(t_{S2})} \times 100$$

Description of the data is given below:

Population 1: [Source: Singh and Mangat (1996), p. 180]

240	n_1=18	n_2=6
	$\overline{X}_1$= 94.35	$\overline{X}_2$=58.46
n=24	$\overline{Y}_1$=98.98	$\overline{Y}_2$=70.6
	C_{y_1} =0.118	C_{y_2}=0.104
	C_{x_1} =0.1298	C_{x_2} =0.0962
	S^2_{x1}=150.134	$S^2_{x_2}$ =31.658
	$S^2_{y_1}$=137.261	$S^2_{y_2}$ =54.84
	S_{yx_1} =136.996	S_{yx_2} =24.084

Population 2: [Source: Singh and Mangat (1996), p. 208]

N=1344	n_1=14	n_2=9	n_3=12	n_4=17
	N_1=400	N_2=216	N_3=364	N_4=364
n=53	$\overline{X}_1$=76.21	$\overline{X}_2$=58.11	$\overline{X}_3$=69.08	$\overline{X}_4$=63.71
	$\overline{Y}_1$=79.35	$\overline{Y}_2$=59.44	$\overline{Y}_3$=76.66	$\overline{Y}_4$=64.57
	$\beta_{21}(x)$=2.22	$\beta_{22}(x)$=2.29	$\beta_{23}(x)$=1.96	$\beta_{24}(x)$=2.47
	C_{x_1} =0.1906	C_{x_2} =0.2416	C_{x3}=0.201	C_{x4}=0.1908
	S^2_{x1}=851.00	$S^2_{x_2}$ =31.06667	$S^2_{x_3}$=35.00	S^2_{x4}=35.00
	$S^2_{y_1}$ =166.70	$S^2_{y_2}$ =174.28	$S^2_{y_3}$ =226.60	S^2_{y4} =170.61
	S_{yx_1} =148.76	S_{yx_3} =161.19	S_{yx_3} =192.21	S_{yx4}=143.83

Population 3: [Source: Singh and Mangat (1996), p. 213]

N=2119	n_1=12	n_2=13	n_3=14
	N_1=640	N_2=710	N_3=769
n=39	$\overline{X}_1$=103.41	$\overline{X}_2$=110.92	$\overline{X}_3$=104.28
	$\overline{Y}_1$=25.75	$\overline{Y}_2$=28.94	$\overline{Y}_3$=25.52
	$\beta_{21}(x)$=2.27	$\beta_{22}(x)$=3.43	$\beta_{23}(x)$=2.89
	C_{x_1} =0.11	C_{x_2} =0.07	C_{x3}=0.11
	S^2_{x1}=133.9	$S^2_{x_2}$ =66.24	$S^2_{x_3}$=154.99
	$S^2_{y_1}$ =40.15	$S^2_{y_2}$ =30.33	$S^2_{y_3}$ =44.42
	S_{yx_1} =67.47	S_{yx_2} =41.03	S_{yx_3} =78.81

Population 4: [Source: Singh and Mangat (1996), p. 219]

N=150 n=25	n_1=6	n_2=1	n_3=7
	N_1=36	N_2=72	N_3=42
	$\bar{X}_1$=366.66	$\bar{X}_2$=310.83	$\bar{X}_3$=317.14
	$\bar{Y}_1$=135	$\bar{Y}_2$=99.16	$\bar{Y}_3$=80.71
	$\beta_{21}(x)$=2.28	$\beta_{22}(x)$=3.26	$\beta_{23}(x)$=3.13
	C_{x_1}=0.1418	C_{x_2}=0.1395	C_{x3}=0.1695
	S_{x1}^2=2706.66	$S_{x_2}^2$=1881.06	$S_{x_3}^2$=2890.47
	$S_{y_1}^2$=80	$S_{y_2}^2$=226.51	$S_{y_3}^2$=120.23
	S_{yx_1}=440	S_{yx_2}=618.93	S_{yx_3}=444

Table 5.1: Percent relative efficiency of $\bar{y}_{st}, \bar{y}_{RS}, t_{\alpha sopt}, t_{S1}$, and t_{S2}

Estimator⇨ Population⇩	$\bar{y}_{st}$	$\bar{y}_{RS}$	$t_{\alpha sopt}$	t_{s1}	t_{s2}
Population 1	100	331.73	393.78	377.21	235.24
Population 2	100	224.85	451.85	324.65	281.51
Population 3	100	331.43	424.64	359.78	298.57
Population 4	100	199.95	748.631	251.46	273.18
Population 5	100	169.27	537.63	345.09	99.99

6. Conclusion:

Table 5.1 reveals that proposed estimator $t_{\alpha s}$ is more efficient than usual unbiased estimator $\bar{y}_{st}$ and the classical separate ratio estimator $\bar{y}_{RS}$. Many estimators can be derived by substituting suitable values of constant α for instance for $\alpha = 1 - f = 1 - \frac{n}{N}$ proposed estimator $t_{\alpha s}$ perform better than aforesaid estimators for all five populations considered and for $\alpha = 1 + f$ it perform better for second and third population. Thus $t_{\alpha s}$ is recommended for use in practice.

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FORECASTING OF AREA, YIELD AND PRODUCTION OF HORSE GRAM IN ODISHA

Balaga Divya, Abhiram Dash*, Akhilesh Kumar Gupta and Manoranjan Sen

Department of Agricultural Statistics,

College of Agriculture, OUAT, Bhubaneshwar

*Corresponding author E-mail: abhidash2stat@gmail.com

Abstract:

Horse gram is one of the important pulse crop of Odisha. It has very high nutritive value and thus contribute towards the nutritional security of the state. Forecasting of horse gram production is very much necessary to enable the agriculture planners to formulate appropriate policies regarding the cultivation of the crop. The present research is carried out on forecasting area, yield and production of horse gram in Odisha by using ARIMA model.

ARIMA, the most widely used model for forecasting is used in the study. The data on area, yield and production of horse gram are collected from 1970-71 to 2019-20 are used to fit the models found suitable from ACF and PACF plots. The ACF and PACF plots are obtained from stationarized data. The best fit model is selected on basis of significance of estimated coefficients, model diagnostic tests and model fit statistics. The selected best fit model is cross validated by refitting the model by leaving last 5 years, 4years, up to last 1 year data and obtaining one step ahead forecast for the years 2015-16 to 2019-20. After successful cross validation the selected best fit model is used for forecasting the area, yield and production of horse gram in Odisha for the future years 2020-21, 2021-22, 2022-23.

The ARIMA model found to be best fit for area, yield and production of horse gram are ARIMA(1,1,0), ARIMA(0,1,1), ARIMA(0,1,1) respectively. All these selected models are fitted without constant as the constant term is insignificant for all these cases. The forecasted values for area under horse gram found to increase in the future years which is responsible for increase in forecasted values of future production despite the yield remaining stagnant for future years.

Keywords: ARIMA, cross validation, forecast, model diagnostics, model fit statistics

1. Introduction:

Pulses are grain legumes which have been major part of the diet and rich source of nutrients. Pulses occupy a privileged position in Odisha agriculture for their contribution to the economy of the state.

Pulses like green gram, cowpea, black gram, arhar, Bengal gram, field pea, horse gram, lentil etc., are grown in the state of Odisha. Among the pulses horse gram on an average occupies 16.57 percent of the total pulse grown area and a production of 14.5 percent. During last 5 years in Odisha, Nabarangpur district stands first in area and production of horse gram with 19.17 thousand hectares and 9.81 thousand MT respectively followed by Balangir and Sundargarh where as Kendrapara district stands first in yield with 4.73 tonnes/ha followed by Jagatsinghpur and Cuttack. Crop area estimation and prediction of crop production and yield beforehand plays an important role in supporting agricultural policy decision making. The forecasting of the production of horse gram is of utmost importance for framing appropriate food policies and ensuring nutritional security of the state.

Various studies have been found under this area of research. Vishwajit et.al (2018) studied about the modelling and forecasting of arhar in major arhar growing states in India using ARIMA and other models. Devgowda S.R. et.al (2019) studied the analysis of variability in area, yield, production and value of pulses in India and Mishra et.al (2021) studied the trend in the production of total pulses in major growing states in India using ARIMA.

2. Materials and Methods:

The secondary data on area, yield and production of horse gram are collected for the state of Odisha (kharif and rabi seasons combined) for the period 1970-71 to 2019-20 from *Five Decades of Odisha Agriculture Statistics* published by Directorate of Agriculture and Food Production, Odisha.
An Autoregressive Integrated Moving Average is a statistical model which is used to predict the future trends. The ARMA models, which includes the order of differencing (which is to stationarize the data) is known as Autoregressive integrated moving average (ARIMA) models. A non-seasonal ARIMA model is classified as an "ARIMA (p,d,q)" model, where, the parameters p,d,q are the non-negative integers where p is the number of autoregressive terms, d is the number of nonseasonal differences necessary for stationarizing the data, and q is the number of moving average terms. Thus, the ARIMA (p,d,q) model can be represented y the following general forecasting equation:

$$Y_t = \mu + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

Where μ is a mean, $\phi_1, \phi_2, \ldots.. \phi_p$ and $\theta_1, \theta_2, \ldots \ldots \theta_j$ are the parameters of the model, p is the order of the autoregressive term, q is the order of the moving average term, and ε_t , $\varepsilon_{t-1}, .. \varepsilon_{t-j}$ are noise error terms.

3. Model identification:

The ARIMA model is fitted to stationary data i.e. having constant mean and variance. Staionarity of data can be tested by using Augmented Dickey-Fuller test. If it is not stationary then it should be converted into stationary series by differencing the data at suitable lag. Usually, the data is stationarized after 1 or 2 differencing. After stationarizing the data, the Auto Correlation Function (ACF) and Partial Auto Correlation Function (PACF) plots are used to identify tentative Auto Regression (AR) and Moving Average (MA) orders. Various tentative models based on identified AR and MA orders are fitted and parameters are estimated. After fitting the tentative models for a variable (area/yield/production) the estimated coefficients are tested for the significance and the normality and independency of the residuals of the fitted models are checked by using Shapiro-Wilk's test statistic and Box-Pierce test statistic respectively. The models having all the estimated coefficients significant and satisfying the normality and independency of the errors are now compared on the basis of model fit statistics like Mean Absolute Percentage Error (MAPE), Root Mean Square Error (RMSE) and Akaike's Information Criteria corrected (AICc). Then the model having the lowest value of these model fit statistics is considered to be the best fit model for the variable.

The model fit statistics like MAPE, RMSE and AICc are mathematically as follows:

Mean absolute percentage error: $\frac{100}{n}\sum_{t=1}^{n}\left|\frac{y_t-\hat{y}_t}{y_t}\right|$

Root mean square error (RMSE): $\sqrt{\frac{\sum_{T=1}^{n}(\hat{y}_t-y_t)^2}{n}}$

where $\hat{y}_t$= forecasted value, y_t= actual value and n = number of times the summation iteration happens

Akaike's information criteria corrected: AIC + $\frac{2K^2+2K}{n-k-1}$

Where AIC is the Akaike's Information criteria, k denotes the number of parameters and n

The model with lowest denotes the sample size. RMSE, MAPE and AICc values is selected as the best fit ARIMA model among selected tentative models and it is taken for forecasting.

4. Results and Discussion:

The data on area, yield and production of horse gram crop was tested for the presence of stationarity by using Augmented Dickey Fuller test and the results are presented in table 1. The test results confirmed that the data was not stationary and made stationary by first order differencing at lag 2.

Table 1: Test of stationarity of data on area, yield and production of horse gram in Odisha

Variable	Original series		First order differenced series	
	ADF test statistic	**P value**	**ADF test statistic**	**P value**
Area	-2.515	0.3694	-3.938	0.0211
Yield	-1.6496	0.7129	-5.0007	0.01
Production	-2.1016	0.5336	-4.1555	0.0105

After stationarising the data the next step is to identify the order of AR and MA terms such as p and q using the ACF and PACF plots of stationary data shown in figures 1, 2 and 3. The ACF plot gives the order of Moving Average and PACF plot about the order of Autoregression. Different tentative models were identified using the orders of AR and MA terms.

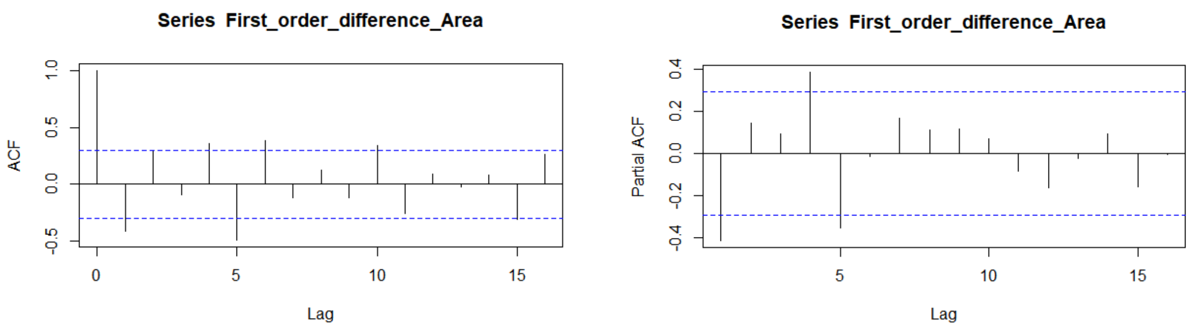

Figure 1: ACF and PACF plot of first order difference of Area under Horse gram

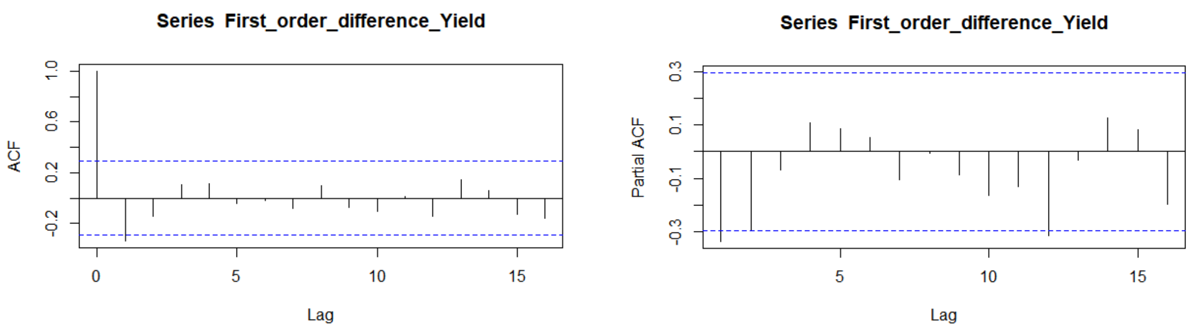

Figure 2: ACF and PACF plot of first order difference of Yield of Horse gram

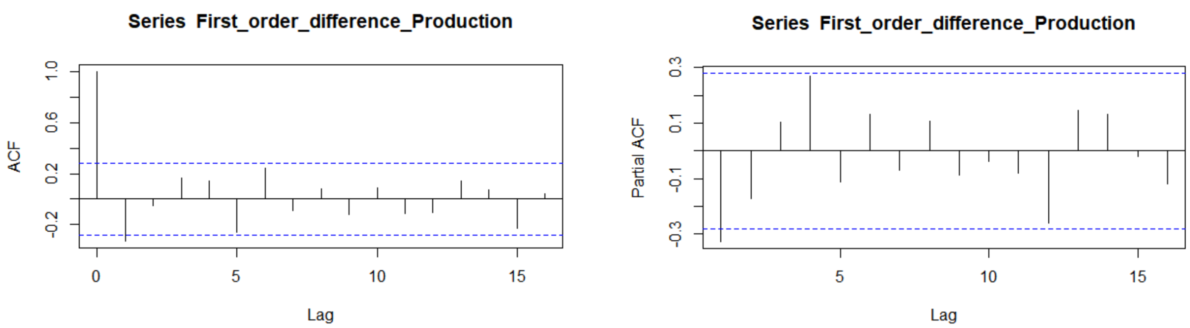

Figure 3: ACF and PACF plot of first order difference of Production of Horse gram

The tentative models of area and their estimated coefficients along with error measures are shown in the table-2. The study of the table reveals that ARIMA(1,1,0) and ARIMA(0,1,2) without constant model has all the estimated coefficients significant

Table 2: Parameter estimates of the ARIMA (p,d,q) model fitted to area under Horse gram

ARIMA(p,d,q)	Constant	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
ARIMA (0,1,2)	**----**	**----**	**----**	**----**	**-0.539** (0.140)**	**0.575** (0.194)**	----
ARIMA (0,1,2)	1.601 (4.330)	----	----	----	-0.531** (0.133)	0.571** (0.189)	----
ARIMA (1,1,0)	**----**	**0.408** (0.129)**	**----**	**----**	**----**	**----**	**----**
ARIMA (1,1,2)	----	0.205 (0.204)	----	----	-0.665** (0.166)	0.6671** (0.2072)	----
ARIMA (0,1,3)	----	----	----	----	-0.428** (0.133)	0.547** (0.140)	0.218 (0.175)

Figures inside the parentheses represents the standard error of the parametric estimates.
'*'- at 5% significance level, '**'- at 1% significance level

Table 3: Model fit statistics of the ARIMA (p,d,q) model fitted to area under Horse gram

ARIMA(p,d,q)	Shapiro-wilk test		Box – pierce test		AICc	RMSE	MAPE
	W	p-value	x-squared	p-value			
ARIMA (0,1,2)	0.951	0.037	0.248	0.618	477.84	29.066	7.016
ARIMA (0,1,2)	0.951	0.038	0.263	0.608	480.087	29.544	7.028
ARIMA (1,1,0)	**0.959**	**0.086**	**0.098**	**0.754**	**480.736**	**28.491**	**7.154**
ARIMA (1,1,2)	0.959	0.081	0.012	0.910	479.331	30.910	7.532
ARIMA (0,1,3)	0.961	0.097	0.013	0.907	478.695	28.769	7.125

W - Shapiro-wilk test statistic x-squared - Box – Pierce test statistic

Table 3 shows the model diagnostics test and model fit statistics for the fitted ARIMA models. ARIMA (1,1,0) model satisfies both the test of normality and independency of residuals. The RMSE, MAPE and AICc are less for ARIMA (1,1,0) without constant model. Thus, this model is selected to be the best fit model for production of horse gram crop. Figure 4 also shows that none of the autocorrelations and partial autocorrelations of residuals are significant. This furthers confirms the selection of the respective best fit models.

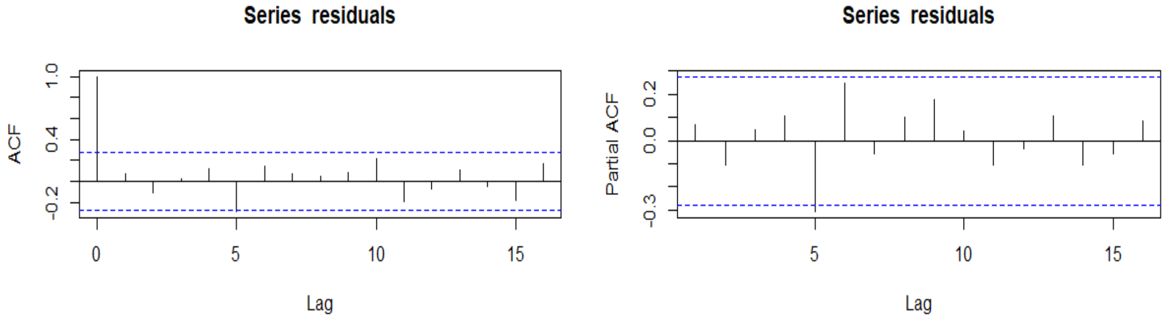

Figure 4: ACF and PACF of residuals from selected ARIMA (1,1,0) model for Area under Horse gram

The tentative ARIMA models of yield and their estimated coefficients along with error measures are shown in the table 4. The study of the table reveals that ARIMA (0,1,1) and ARIMA (2,1,0) without constant model has all the estimated coefficients significant

Table 4: Parameter estimates of the ARIMA (p,d,q) model fitted to area under Horse gram

ARIMA(p,d,q)	**Constant**	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
ARIMA **(0,1,1)**	----	----	----	----	-0.424** (0.122)	----	----
ARIMA (2,1,0)	----	- 0.406** (0.137)	- 0.268* (0.136)	----	----	----	----
ARIMA (0,1,2)	----	----	----	----	-0.409** (0.162)	-0.021 (0.164)	----
ARIMA (0,1,1)	-0.521 (4.169)	----	----	----	0.424** (0.122)	----	----
ARIMA (1,1,1)	----	-0.441* (0.227)	----	----	0.024 (0.269)	----	----

Where, Figure inside the parentheses represents the standard error of the parametric estimators.

'*'- at 5% significance level, '**'- at 1% significance level

Table 5 shows the model diagnostics test and model fit statistics for the fitted ARIMA models for yield of horse gram. ARIMA (0,1,1) without constant model satisfies both the test of normality and independency of residuals. The RMSE, MAPE and AICc are less for ARIMA (0,1,1) without constant model. Thus, this model is selected to be the best fit model for production of horse gram crop. Figure 5 also shows that none of the autocorrelations and partial autocorrelations of residuals are significant. This furthers confirms the selection of the respective best fit models.

Thus this model is selected to be the best fit model for yield under horse gram crop.

Table 5: Model fit statistics of the ARIMA (p,d,q) model fitted to yield of Horse gram

ARIMA(p,d,q)	Shapiro-wilk test		Box – pierce test		AICc	RMSE	MAPE
	W	p-value	x-squared	p-value			
ARIMA (0,1,1)	**0.927**	**0.004**	**0.0003**	**0.985**	**526.817**	**49.458**	**9.021**
ARIMA (2,1,0)	0.936	0.009	0.018	0.890	527.906	48.835	9.092
ARIMA (0,1,2)	0.929	0.005	0.003	0.954	529.072	49.452	9.026
ARIMA (0,1,1)	0.927	0.004	0.0005	0.981	529.074	49.450	9.051
ARIMA (1,1,1)	0.928	0.004	0.0002	0.987	529.081	49.455	9.024

W - Shapiro-wilk test statistic x-squared - Box – pierce test statistic

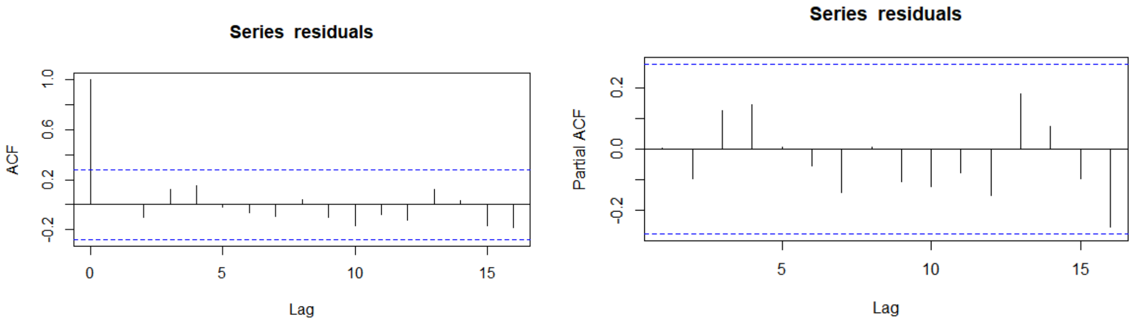

Figure 5: ACF and PACF of residuals from selected ARIMA (0,1,1) model for Yield under Horse gram

The tentative models of production and their estimated coefficients along with error measures are shown in the table 6. The study of the table reveals that ARIMA (0,1,1) and ARIMA (1,1,0) without constant model has all the estimated coefficients significant

Table 6: Parameter estimates of the ARIMA (p,d,q) model fitted to production of Horse gram

ARIMA(p,d, q)	Constant	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
ARIMA (0,1,1)	**----**	**----**	**---**	**---**	**-0.328**** **(0.116)**	**----**	**----**
ARIMA (1,1,0)	----	-0.322* (0.133)	---	---	----	----	----
ARIMA (0,1,2)	----	----	---	---	-0.421* (0.171)	0.137 (0.176)	----
ARIMA (0,1,3)	----	----	---	---	-0.394** (0.138)	0.061 (0.141)	0.162 (0.110)
ARIMA (1,1,1)	----	-0.130 (0.278)	----	---	-0.230 (0.252)	----	----

Figure inside the parentheses represents the standard error of the parametric estimators.

'*'- at 5% significance level, '**'- at 1% significance level

Table 7 shows the model fit statistics and model diagnostics test for the fitted ARIMA models for production of horse gram. ARIMA (0,1,1) and ARIMA (1,1,0) without constant model satisfies both the test of normality and independency of residuals. The RMSE, MAPE and AICc are less for ARIMA (0,1,1) without constant model. Thus, this model is selected to be the best fit model for production of horse gram crop. Figure 6 also shows that none of the autocorrelations and partial autocorrelations of residuals are significant. This furthers confirms the selection of the respective best fit models.

Table 7: Model fit statistics of the ARIMA (p,d,q) model fitted to production of Horse gram

ARIMA(p,d,q)	Shapiro-wilk test		Box – pierce test		AICc	RMSE	MAPE
	W	p-value	x-squared	p-value			
ARIMA (0,1,1)	**0.928**	**0.004**	**0.467**	**0.494**	**454.232**	**23.602**	**14.267**
ARIMA (1,1,0)	0.921	0.003	0.001	0.973	454.729	23.723	14.354
ARIMA (0,1,2)	0.920	0.002	0.021	0.885	455.886	23.436	14.185
ARIMA (0,1,3)	0.932	0.007	0.004	0.950	456.265	22.946	14.370
ARIMA (1,1,1)	0.921	0.002	0.009	0.922	456.301	23.549	14.357

W - Shapiro-wilk test statistic x-squared - Box – Pierce test statistic

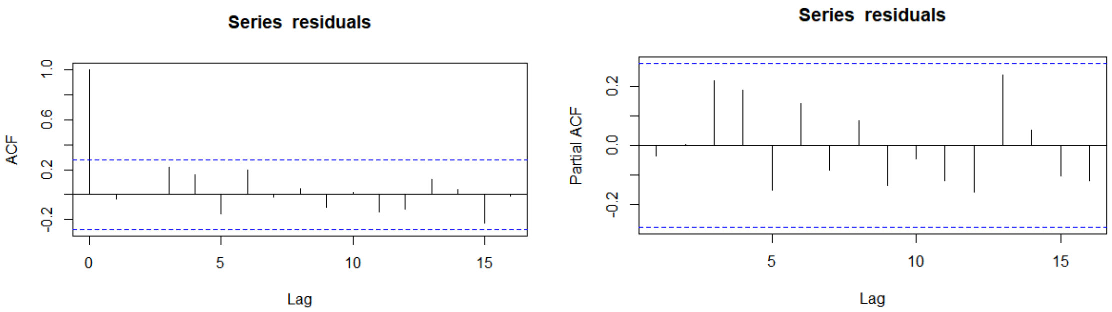

Figure 6: ACF and PACF of residuals from selected ARIMA (0,1,1) model for Production of horse gram

In the table 8, the result of cross validation of the selected best fit ARIMA model by one-step ahead forecasting has been presented. The APE (absolute percentage error) of area under horse gram is found to be in the range between 1 to 19 and the MAPE(mean APE) is found to be 8.912 for area of horse gram crop. Similarly for yield the APE range is found between 0 to 12 and MAPE is 4.844 and for production, APE range is between 1 to 19 and MAPE is 9.946. These results show that the selected ARIMA models are successfully cross validated.

Table 8: Cross validation of selected ARIMA models for area, yield and production of horse gram in Odisha

Year	Area			Yield			Production		
	Actual	Predicted	APE	Actual	Predicted	APE	Actual	Predicted	APE
2015-16	194.96	231.81	18.89	387	384.02	0.77	75.45	89.35	18.43
2016-17	216.2	212.09	1.90	436	385.68	11.54	94.26	79.96	15.16
2017-18	200.01	190.85	4.57	457	413.77	9.45	91.40	89.573	1.99
2018-19	205.25	200.65	2.23	448	438.81	2.05	91.95	90.79	1.25
2019-20	235.67	195.73	16.94	446	444.19	0.41	105.11	91.57	12.88
MAPE	8.912			4.844			9.946		

The appropriate ARIMA models which are represented in the previous tables were used to forecast the area, yield and production of horse gram crop in Odisha for the years 2020-21, 2021-22 and 2022-23.

Figures 7, 8 and 9 shows the actual, fitted and forecast values of area, yield and production of horse gram in Odisha.

Table 9: Forecast values of horse gram in Odisha for the year 2020-21 to 2022-23

Year	Area ('000ha)			Yield (kg/ha)			Production ('000tonnes)		
	Forecasted	95 % confidence interval		Forecasted	95 % confidence interval		Forecasted	95 % confidence interval	
		Lower CI	Upper CI		Lower CI	Upper CI		Lower CI	Upper CI
2020-21	223.23	164.38	282.08	445.23	346.31	544.17	98.66	53.45	147.88
2021-22	241.43	176.35	306.51	445.23	331.07	559.40	100.66	43.78	157.54
2022-23	241.43	152.11	330.75	445.23	317.65	572.82	100.66	35.54	165.79

CI – Class Interval

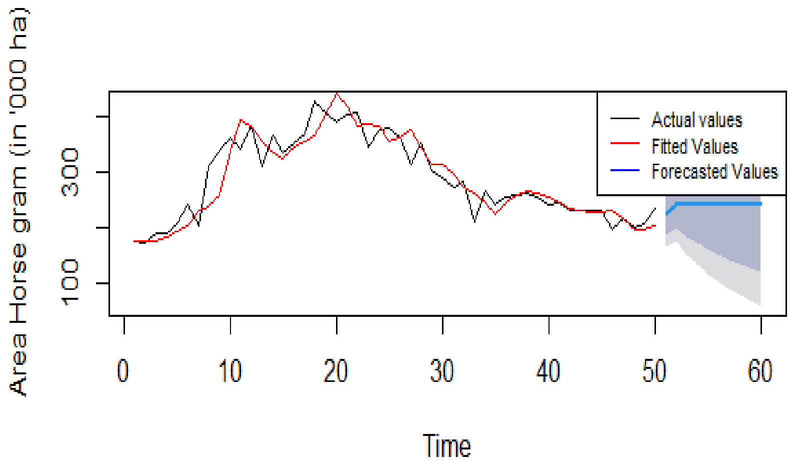

Figure 7: Actual with fitted and forecasted values of Area under horse gram from ARIMA (1,1,0) without constant model

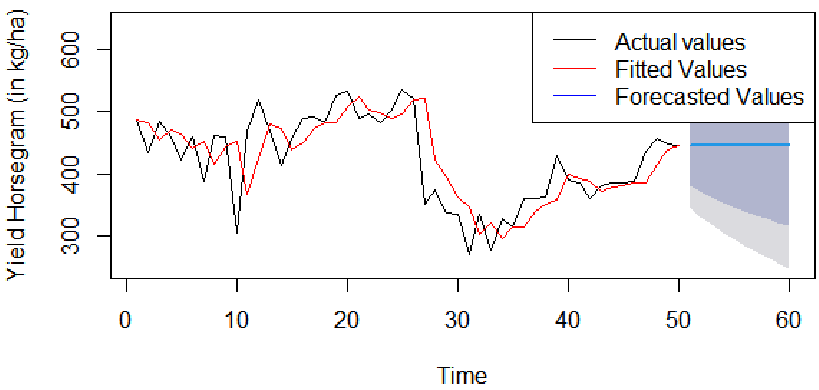

Figure 8: Actual with fitted and forecasted values of Yield of horse gram from ARIMA (0,1,1) without constant model

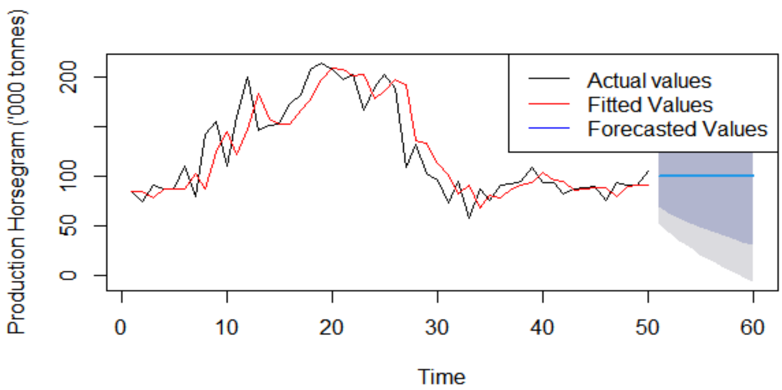

Figure 9: Actual with fitted and forecasted values of Production under horse gram from ARIMA (0,1,1) without constant model

Conclusion:

ARIMA (1,1,0) without model, ARIMA (0,1,1) without constant model and ARIMA (0,1,1) without constant model are found to be the best fit model for area, yield and production of horse gram in Odisha. These selected models are used for forecasting of area, yield and production of horse gram in Odisha. The forecast values shows that area, yield and production of horse gram in Odisha remain stagnant in future years with variation in lower and upper class interval of the forecast values.

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MHD FLOW WITH POROUS MEDIUM

Hariom Singh Tomar*[1] and Pradeep Kashyap[2]

[1]Department of Mathematics,

Chintamani College of Arts and Science,

Gondpipri, Dist- Chandrpur (M.S.)-442702

[2]Department of Mathematics,

Shri Sadguru Saibaba Science and Commerce College,

Ashti, Dist- Gadchiroli (M.S.)-442707

*Corresponding author E-mail: hstomar5@gmail.com

This chapter means Magneto hydrodynamic, in simple, sense the motion of the electrical conducting fluid in the presence of magnetic field. Due to the effect of magnetic field there is created in electrical current in the fluid which modifies the fluid motion.

The theory of electric conductivity implies that there are electric charges in motion in the fluid. The motion of charges generates a magnetic field which exerts a force on a charged particle of fluid moving within the fluid. Such forces are generated by basic laws of electrically and magnetism.

The basic equations known as Maxwell's equation $\vec{J}$ and $\vec{B}$ are given by Maxwell's equations and Ohms law namely $= -\sigma \vec{u} B^2$

$$\text{Curl } \vec{H} = 4\pi \vec{J}, \ \nabla.E = \frac{4\pi}{\in} q, \ \nabla.\vec{E} = -\mu \frac{\partial H}{\partial t}$$

$$\text{div } \vec{B} = 0, \ \nabla \vec{H} = 0, \ \text{Curl } \vec{E} = 0, \ \nabla \times \vec{E} = -H_0 \frac{\partial H}{\partial t}$$

$$\vec{J} = \sigma[\vec{E} + (\vec{u} \times \vec{B})] \quad (1)$$

where $\vec{E}$ is the electrical field intensity, $\in$ is dielectric constant of medium, $\vec{B} = (\mu_0 \vec{H})\vec{H}$ is a magnetic field intensity, μ_0 is the permeability, $\vec{J}$ is conjuction current density vector, q is the charge per unit volumes and σ is conductivity of the equation. $\vec{U}$ is fluid velocity vector, $\vec{B}$ the magnetic field.

It is assumed that the effect of the induced magnetic field and electric field produced by the motion of electrically conducting liquid is negligible and no electric field is applied. With these assumptions the magnetic term $\vec{J} \times \vec{B}$ of body is given by

$$\vec{J} \times \vec{B} = \sigma [\vec{E} + (\vec{U} \times \vec{B})] \times \vec{B} \qquad (2)$$

$$\vec{J} \times \vec{B} = \sigma[\vec{U} \times \vec{B}] \times \vec{B}(\text{as E is absent}) \qquad (3)$$

$$\vec{J} \times \vec{B} = \sigma[\vec{B}(\vec{U}.\vec{B}) - \vec{U}(\vec{B}.\vec{B})] \qquad (4)$$

$$\vec{J} \times \vec{B} = -\sigma\vec{u}B^2 \qquad (5)$$

Flow through porous media:

In recent years, the study of flows through porous media has been causing keen interest amongst the engineers and the mathematicians due to its importance and video applications in the fields of petroleum technology, soil mechanics, ground water hydrology, seepage of water in river beds, purification and filtration process and bio-mechanics etc.

The porous mateiral containing the field is the fact a non-homogeneous medium but for the suke of anlaysie, it may be possible to replce it with a homogeneous liuqid which has dynamical poperties equal to the local averages of the original non-homogeneous continum. Then one can study the flow of a hypothetical homogeneous fluid under the action of the property average of external forces. Hence, a complicated problem of the flow through a porous medium reduces to the flow problem of a homogeneous fluid with some additinol resistance.

Flow of homogeneous fluid through various types of porous media were presented by Muskat (1946) following the classical Daracy's experimental law, which state that seepage velocity of the fluid is proportional to the pressure gradient. This law fails to explain the phenomenon occurring in highly porous media. The viscous stress at the surface is able to penetrate into the medium and produce flow near the surface even in the absence of pressure gradient.

Brinkman (1947) proposed a general Daracy's law to study flow through highly porous media

$$0 = -\nabla\vec{p} + \mu\nabla^2\bar{v} = \frac{\mu}{K}\bar{v} \qquad (A)$$

where $\bar{v}$ and $\bar{p}$ represent velocity and pressure field, m is the velocity coefficient of fluid and K is the permeability constant of the medium. Later Tam (1969) derived analytically the same equations to study the flow past spherical particles at low Reynolds number. Yamamoto (1971, 73) and Gulab Ram and Mishra (1977) examined the flow past porous bodies applying the generalized law.

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DIMENSIONAL ANALYSIS AND SIMILITUDE

Hariom Singh Tomar*[1] and Pradeep Kashyap[2]

[1]Department of Mathematics,

Chintamani College of Arts and Science,

Gondpipri, Dist- Chandrpur (M.S.)-442702

[2]Department of Mathematics,

Shri Sadguru Saibaba Science and Commerce College,

Ashti, Dist- Gadchiroli (M.S.)-442707

***Corresponding author E-mail: hstomar5@gmail.com**

This chapter has considered fundamentals of dimensional analysis and similitude, which are commonly used in experimental fluid mechanics. The viscous, incompressible fluids for which the velocity components of fluid and the pressure p flow the Navier-Stokes equations:

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k^2} \tag{1}$$

and $$\frac{\partial u_k}{\partial x_k} = 0 \tag{2}$$

Where, k is a dummy suffix and i, k = 1, 2 or 3.

The energy equation is,

$$\rho c_p \left(\frac{\partial T}{\partial t} + u_k \frac{\partial T}{\partial x_k} \right) = K \frac{\partial^2 T}{\partial x_k^2} + \mu \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) \frac{\partial u_i}{\partial x_k} \tag{3}$$

where, c_p is specific heat at constant volume, K is coefficient of thermal conductivity, μ is coefficient of viscosity and T is temperature.

To bring the essential parameters of laminar flow, the fundamental equations stated earlier are transformed by introducing the non-dimensional quantities:

$$\left. \begin{array}{l} t' = \dfrac{t}{t_0},\ x_i' = \dfrac{x_i}{\rho_0}, u_i' = \dfrac{u_i}{u_0}, p' = \dfrac{p}{\rho u_0^2},\ T' = \dfrac{T}{T_0}, \\ R = \dfrac{u_0 \rho_0}{\nu},\ N_u = \dfrac{h\rho_0}{K},\ P_r = \dfrac{c_p \mu}{K} \text{ and } E_c = \dfrac{u_0^2}{T_0 c_p} \end{array} \right\} \tag{4}$$

Where, ℓ_0, u_0, t_0 and T_0 are respectively characteristic length, velocity, time and temperature and R, N_u and E_c are Reynolds number, Nusselt number, Prandtl number and Eckert number respectively.

Reynolds number R measures the ratio of inertial force to the viscous force. For small R, the viscous force is predominant and effect of viscosity is signiticant in the flow field and for large R, inertial force dominates and effect of viscosity is important only in the narrow boundary layer region near the solid boundary or in the region with large variation in velocity. Nusselt number is a measure of the convective heat transfer and the Prandtl number is a measure of relative importance of viscosity and heat conduction. It can be written as:

$$P_r = \frac{\nu}{K / c_p \rho} = \frac{\text{Kinematic visos ity}}{\text{Thermal diffusivit y}} \quad (5)$$

The two flows are similar if they are geometrically similar and if all the relevant dimensionless parameters are the same for both flows. To summarize, we model as follows for incompressible and compressible flows:

R the same	:	viscous flow and subsonic aerodynamics,
R and K the same	:	high speed compressible and supersonic flow,
R and $M_0$the same	:	compressible boundary layer,
P_r the same	:	heat conduction,
R, M_0 and P_r the same:		compressible boundary layer with heat conduction,
R and F_r the same	:	Marine ship modeling,

Where, R, M_0, P_r, F_r and K are Reynold's number, Mach number, Prandtl number, Froude number and ratio of specific heats respectively.

Dimensional analysis is very useful for planning, presentation, and interpretation of experimental data. As discussed previously, most practical fluid mechanics problems are too complex to solve analytically and must be tested by experiment or approximated by computational fluid dynamics (CFD). These data have much more generality if they are expressed in compact, economic non-dimensional form. Dimensional analysis is a method for reducing the number and complexity of experimental variables that affect a given physical phenomena.

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SEPARATE ESTIMATORS IN STRATIFIED RANDOM SAMPLING

Hilal A. Lone[1], Rajesh Tailor[2] and Med Ram Verma[3]

[1]Department of Statistics, Government Degree College, Baramulla, J&K

[2]School of Studies in Statistics, Vikram University Ujjain, M.P

[3]Division of Livestock Economics, Statistics and Information Technology,

ICAR-IVRI, Izatnagar, Bareilly, U.P.

Abstract:

This article suggests two separate ratio-cum-product type estimators for estimating the finite population mean in stratified random sampling. The efficiency of the suggested estimators are compared with the usual unbiased estimator in stratified random sampling, conventional separate ratio and product estimators and Tailor and Lone (2014) separate ratio and product type estimators on the basis of mean square error (MSE). The bias and mean square error of considered estimators are obtained. The results are illustrated by three data sets.

Keywords: Finite population mean, separate ratio estimator, Auxiliary variable, Bias, Mean squared error.

1. Introduction:

When information of parameters of auxiliary variate is available in each stratum, separate ratio type estimators may be constructed easily and perform better as compared to combined estimators.

In this chapter, separate ratio-cum -product estimators for finite population mean are suggested using known parameters of auxiliary variates in each stratum.

Let as consider finite population U which is divided into L strata of sizes $N_h \{h = 1,2,3,..., L\}$. From each stratum a sample of size n_h is drawn using simple random sampling without replacement such that $n = \sum_{h=1}^{L} n_h$.

The classical separate ratio and product estimators are defined respectively as

$$t_1 = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h} \right) \tag{1.1}$$

and

$$t_2 = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{x}_h}{\bar{X}_h} \right) . \tag{1.2}$$

Upadhyaya and Singh (1999) used coefficient of kurtosis $\beta_2(x)$ and coefficient of variation C_x of auxiliary variate x and defined two different ratio type estimators as

$$t_3 = \bar{y}\left(\frac{\bar{X}\ \beta_2(x) + C_x}{\bar{x}\,\beta_2(x) + C_x}\right) \tag{1.3}$$

and

$$t_4 = \bar{y}\left(\frac{\bar{X}\,C_x + \beta_2(x)}{\bar{x}\,C_x + \beta_2(x)}\right) . \tag{1.4}$$

Kadilar and Cingi (2003) defined Upadhyaya and Singh (1999) estimators in stratified random sampling as

$$t_5 = \bar{y}_{st}\ \frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h\,\beta_{2h}(x) + C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h\,\beta_{2h}(x) + C_{xh}\right)} \tag{1.5}$$

and

$$t_6 = \bar{y}_{st}\ \frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h\,C_{xh} + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h\,C_{xh} + \beta_{2h}(x)\right)} . \tag{1.6}$$

Tailor and Lone (2014) suggested separate ratio type estimators in stratified random sampling as

$$t_7 = \sum_{h=1}^{L} W_h\bar{y}_h\ \left\{\frac{\bar{X}_h\beta_{2h}(x) + C_{xh}}{\bar{x}_h\beta_{2h}(x) + C_{xh}}\right\} \tag{1.7}$$

and

$$t_8 = \sum_{h=1}^{L} W_h\bar{y}_h\left\{\frac{\bar{X}_h C_{xh} + \beta_{2h}(x)}{\bar{x}_h C_{xh} + \beta_{2h}(x)}\right\} . \tag{1.8}$$

Product version of $\hat{\bar{Y}}_{RS}^{US1}$ and $\hat{\bar{Y}}_{RS}^{US2}$ can be defined as

$$t_9 = \sum_{h=1}^{L} W_h\bar{y}_h\ \left\{\frac{\bar{x}_h\beta_{2h}(x) + C_{xh}}{\bar{X}_h\beta_{2h}(x) + C_{xh}}\right\} \tag{1.9}$$

and

$$t_{10} = \sum_{h=1}^{L} W_h\bar{y}_h\left\{\frac{\bar{x}_h C_{xh} + \beta_{2h}(x)}{\bar{X}_h C_{xh} + \beta_{2h}(x)}\right\} . \tag{1.10}$$

Mean squared error of the t_1 , t_2 , t_7 , t_8 , t_9 and t_{10}

$$MSE\left(t_1\right)=\sum_{h=1}^{L} W_h^2\,\gamma_h\left(S_{yh}^2+R_{1h}^2\,S_{xh}^2\;-2\,R_{1h}S_{yxh}\right), \tag{1.11}$$

$$MSE\left(t_2\right)=\sum_{h=1}^{L} W_h^2\,\gamma_h\left(S_{yh}^2+R_{1h}^2\,S_{xh}^2\;+2\,R_{1h}S_{yxh}\right), \tag{1.12}$$

$$MSE\left(t_7\right)=\sum_{h=1}^{L} W_h^2\,\gamma_h\left(S_{yh}^2+R_{1h}^2\lambda_{3h}^2\,S_{xh}^2\;-2\,R_{1h}\,\lambda_{3h}\,S_{yxh}\right), \tag{1.13}$$

$$MSE\left(t_8\right)=\sum_{h=1}^{L} W_h^2\,\gamma_h\left(S_{yh}^2+R_{1h}^2\lambda_{3h}^2\,S_{xh}^2\;+2\,R_{1h}\,\lambda_{3h}\,S_{yxh}\right), \tag{1.14}$$

$$MSE\left(t_9\right)=\sum_{h=1}^{L} W_h^2\,\gamma_h\left(S_{yh}^2+R_{1h}^2\lambda_{4h}^2\,S_{xh}^2\;-2\,R_{1h}\,\lambda_{4h}\,S_{yxh}\right), \tag{1.15}$$

$$MSE\left(t_{10}\right)=\sum_{h=1}^{L} W_h^2\,\gamma_h\left(S_{yh}^2+R_{1h}^2\lambda_{4h}^2\,S_{xh}^2\;+2\,R_{1h}\,\lambda_{4h}\,S_{yxh}\right), \tag{1.16}$$

where

$$\lambda_{3h}=\frac{\overline{X}_h\,\beta_{2h}(x)}{\overline{X}_h\,\beta_{2h}(x)+C_{xh}}\quad,\ \lambda_{4h}=\frac{\overline{X}_h\,C_{xh}}{\overline{X}_h\,C_{xh}+\beta_{2h}(x)}\quad \text{and}\ \ \rho_{yzh}=\frac{S_{yxh}}{S_{yh}\,S_{xh}}\ .$$

2. Suggested separate ratio - cum - producrt estimtors

Assuming that coefficient of variation C_{xh} and coefficient of kurtosis $\beta_{2h}(x)$ in h^{th} stratum are know in advance with stratum mean, suggested separate ratio - cum -product estimators for population mean are

$$\hat{\overline{Y}}_{RPS}^{US1}\;=\;\sum_{h=1}^{L} W_h\,\overline{y}_h\left(\frac{\overline{X}_h\,\beta_{2h}(x)+C_{xh}}{\overline{x}_h\,\beta_{2h}(x)+C_{xh}}\right)\left(\frac{\overline{z}_h\,\beta_{2h}(z)+C_{zh}}{\overline{Z}_h\;\beta_{2h}(z)+C_{zh}}\right) \tag{2.1}$$

and

$$\hat{\overline{Y}}_{RPS}^{US2}\;=\sum_{h=1}^{L} W_h\,\overline{y}_h\left(\frac{\overline{X}_h\,C_{xh}+\beta_{2h}(x)}{\overline{x}_h\,C_{xh}+\beta_{2h}(x)}\right)\left(\frac{\overline{z}_h\,C_{zh}+\beta_{2h}(z)}{\overline{Z}_h\,C_{zh}+\beta_{2h}(z)}\right) \tag{2.2}$$

To obtain the bias and mean Squared error of the suggested estimator $\hat{\overline{Y}}_{RPS}^{US1}$, we write

$\overline{y}_h=\overline{Y}_h\,\left(1+e_{oh}\right),\overline{x}_h=\overline{X}_h\left(1+e_{1h}\right)$ and $\overline{z}_h=\overline{Z}_h\,\left(1+e_{2h}\right)$

Such that $E\left(e_{oh}\right)=E\,\left(e_{1h}\right)=E\left(e_{2h}\right)=0$ and

$E\left(e_{oh}^2\right)=\gamma_h\;C_{yh}^2$,

$E\left(e_{1h}^2\right)=\gamma_h\,C_{xh}^2$,

$E\left(e_{oh}\;e_{1h}\right)=\gamma_h\,\rho_{yxh}\;C_{yh}\;C_{xh}$,

$E(e_{1h}\ e_{2h}) = \gamma_h\ \rho_{xzh}\ C_{xh}\ C_{zh}$,

$E\ (e_{oh}\ e_{2h}) = \gamma_h\ \rho_{yzh}\ C_{yh}\ C_{zh}$,

Now expressing (2.1) in terms of $e_{ih}'s$ we have

$$\hat{\bar{Y}}_{RPS}^{US1} = \sum_{h=1}^{L} W_h\ \bar{Y}_h\ (1+e_{oh})\ (1-\lambda_{3h}\ e_{1h} + \lambda_{3h}^2\ e_{1h}^2)\ (1+\mu_{3h}\ e_{2h})$$

Finally, the bias and mean squared error of the suggested estimator $\hat{\bar{Y}}_{RPS}^{US1}$ up to the first degree of approximation are obtained as

$$B\left(\hat{\bar{Y}}_{RPS}^{US1}\right) = \sum_{h=1}^{L} W_h \bar{Y}_h \gamma_h\ \left(\lambda_{3h}^2\ C_{xh}^2 - \lambda_{3h}\ \mu_{3h}\ \rho_{xzh}\ C_{xh}\ C_{zh} + \mu_{zh}\ \rho_{yzh}\ C_{yh}\ C_{zh} \right.$$
$$\left. -\lambda_{3h}\ \rho_{yxh}\ C_{yh}\ C_{xh}\right) \tag{2.3}$$

$$MSE\left(\hat{\bar{Y}}_{RPS}^{US1}\right) = \sum_{h=1}^{L} W_h^2\ \gamma_h\ \left(S_{yh}^2 + \mu_{3h}^2\ S_{zh}^2\ R_{2h}^2 + \lambda_{3h}^2\ S_{xh}^2\ R_{1h}^2 + 2\mu_{3h}\ R_{2h}\ S_{yzh} \right.$$
$$\left. -2\,\mu_{3h}\ \lambda_{3h}\ R_{1h}\ R_{2h}\ S_{xzh} - 2\lambda_{3h}\ R_{1h}\ S_{yxh}\right) \tag{2.4}$$

where

$$\lambda_{3h} = \frac{\bar{X}_h\ \beta_{2h}(x)}{\bar{X}_h\ \beta_{2h}(x) + C_{xh}} \quad \textit{and} \quad \mu_{3h} = \frac{\bar{Z}_h\ \beta_{2h}(z)}{\bar{Z}_h\ \beta_{2h}(z) + C_{zh}}\ .$$

Similarly, the bias and mean squared error of the suggested estimator $\hat{\bar{Y}}_{RPS}^{US2}$ up to the first degree of approximation are obtained as

$$B\left(\hat{\bar{Y}}_{RPS}^{US2}\right) = \sum_{h=1}^{L} W_h\ \bar{Y}_h \gamma_h\ \left(\lambda_{4h}^2\ C_{xh}^2 - \lambda_{4h}\ \mu_{4h}\ \rho_{xzh}\ C_{xh}\ C_{zh} - \lambda_{4h}\ \rho_{yxh}\ C_{yh}\ C_{xh} \right.$$
$$\left. +\ \mu_{4h}\ \rho_{yzh}\ C_{yh}\ C_{zh}\right) \tag{2.5}$$

$$MSE\left(\hat{\bar{Y}}_{RPS}^{US2}\right) = \sum_{h=1}^{L} W_h^2\ \gamma_h\ \left(S_{yh}^2 + \mu_{4h}^2\ S_{zh}^2\ R_{2h}^2 + \lambda_{4h}^2\ S_{xh}^2\ R_{1h}^2 + 2\,\mu_{4h}\ S_{yzh}\ R_{2h} \right.$$
$$\left. -2\,\mu_{4h}\ \lambda_{4h}\ S_{xzh}\ R_{1h}\ R_{2h} - 2\lambda_{4h}\ S_{yxh}\ R_{1h}\right) \tag{2.6}$$

where $\lambda_{4h} = \dfrac{\bar{X}_h\ C_{xh}}{\bar{X}_h\ C_{xh} + \beta_{2h}(x)}$ *and* $\mu_{4h} = \dfrac{\bar{Z}_h\ C_{zh}}{\bar{Z}_h\ C_{zh} + \beta_{2h}\ (z)}$

3. Efficiency comparisons

Variance of the unbiased estimator $\bar{y}_{st}$ is

$$V\left(\bar{y}_{st}\right)=\sum_{h=1}^{L} W_h^2\ \gamma_h S_{yh}^2 \tag{3.1}$$

From (3.1), (1.11), (1.12), (1.13), (1.14) and (2.4), it is observed that the suggested estimator $\hat{\hat{Y}}_{RPS}^{US1}$ would be more efficient than

(i) $\bar{y}_{st}$ if

$$\sum_{h=1}^{L} W_h^2\,\gamma_h\left(\mu_{3h}^2\, S_{zh}^2 R_{2h}^2 + \lambda_{3h}^2\ S_{xh}^2\ R_{1h}^2 + 2\,\mu_{3h}\ R_{2h}\ S_{yzh} - 2\,\mu_{3h}\,\lambda_{3h}\,R_{1h}\,R_{2h}\,S_{xzh} - 2\lambda_{3h}\ R_{1h}\ S_{yxh}\right) < 0 \tag{3.2}$$

(ii) t_1 if

$$\sum_{h=1}^{L} W_h^2\,\gamma_h\left(\mu_{3h}^2\, S_{zh}^2 R_{2h}^2 + \lambda_{3h}^2\ S_{xh}^2\ R_{1h}^2 + 2\,\mu_{3h}\ R_{2h}\ S_{yzh} - 2\,\mu_{3h}\,\lambda_{3h}\,R_{1h}\,R_{2h}\,S_{xzh} - 2\,\lambda_{3h}\ R_{1h}\,S_{yxh}\right) - \sum_{h=1}^{L} W_h^2\,\gamma_h\ \left(S_{xh}^2\ R_{1h}^2 - 2\,R_{1h}\,S_{yxh}\right) < 0 \tag{3.3}$$

(iii) t_2 if

$$\sum_{h=1}^{L} W_h^2\,\gamma_h\left(\mu_{3h}^2\, S_{zh}^2 R_{2h}^2 + \lambda_{3h}^2\ S_{xh}^2\ R_{1h}^2 + 2\,\mu_{3h}\ R_{2h}\ S_{yzh} - 2\,\mu_{3h}\,\lambda_{3h}\,R_{1h}\,R_{2h}\,S_{xzh} - 2\,\lambda_{3h}\ R_{1h}\,S_{yxh}\right) - \sum_{h=1}^{L} W_h^2\,\gamma_h\ \left(S_{xh}^2\ R_{1h}^2 + 2\,R_{1h}\,S_{yxh}\right) < 0 \tag{3.4}$$

(iv) t_7 if

$$\sum_{h=1}^{L} W_h^2\,\gamma_h\left(\mu_{3h}^2\, S_{zh}^2 R_{2h}^2\ + 2\,\mu_{3h}\ R_{2h}\ S_{yzh} - 2\,\mu_{3h}\,\lambda_{3h}\,R_{1h}\,R_{2h}\,S_{xzh}\ < 0 \tag{3.5}$$

(v) t_8 if

$$\sum_{h=1}^{L} W_h^2\,\gamma_h\left(\mu_{3h}^2\, S_{zh}^2 R_{2h}^2 + 2\,\mu_{3h}\ R_{2h}\ S_{yzh} - 2\,\mu_{3h}\,\lambda_{3h}\,R_{1h}\,R_{2h}\,S_{xzh} - 4\lambda_{3h}\ R_{1h}\,S_{yxh}\right)\ <\ 0 \tag{3.6}$$

From (3.1), (1.10), (1.11),(1.15),(1.16) and (2.6) it is observed that the suggested estimator $\hat{\hat{Y}}_{RPS}^{US2}$ would be more efficient than

(i) $\bar{y}_{st}$ if

$$\sum_{h=1}^{L} W_h^2\,\gamma_h\left(\mu_{4h}^2\, S_{zh}^2 R_{2h}^2 + \lambda_{4h}^2\ S_{xh}^2\ R_{1h}^2 + 2\,\mu_{3h}\ R_{2h}\ S_{yzh} - 2\,\mu_{4h}\,\lambda_{4h}\,R_{1h}\,R_{2h}\,S_{xzh} - 2\lambda_{4h}\ R_{1h}\ S_{yxh}\right) < 0 \tag{3.7}$$

(ii) t_1 if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \Big(\mu_{4h}^2 S_{zh}^2 R_{2h}^2 + \lambda_{4h}^2 S_{xh}^2 R_{1h}^2 + 2\mu_{4h} R_{2h} S_{yzh} - 2\mu_{4h} \lambda_{4h} R_{1h} R_{2h} S_{xzh}$$

$$- 2\lambda_{4h} R_{1h} S_{yxh} \Big) - \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{xh}^2 R_{1h}^2 - 2R_{1h} S_{yxh} \right) < 0 \qquad (3.8)$$

(iii) t_2 if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \Big(\mu_{4h}^2 S_{zh}^2 R_{2h}^2 + \lambda_{4h}^2 S_{xh}^2 R_{1h}^2 + 2\mu_{4h} R_{2h} S_{yzh} - 2\mu_{4h} \lambda_{4h} R_{1h} R_{2h} S_{xzh}$$

$$- 2\lambda_{4h} R_{1h} S_{yxh} \Big) - \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{xh}^2 R_{1h}^2 + 2R_{1h} S_{yxh} \right) < 0 \qquad (3.9)$$

(iv) t_9 if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \Big(\mu_{4h}^2 S_{zh}^2 R_{2h}^2 + 2\mu_{4h} R_{2h} S_{yzh} - 2\mu_{4h} \lambda_{4h} R_{1h} R_{2h} S_{xzh} < 0 \qquad (3.10)$$

(v) t_{10} if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \Big(\mu_{4h}^2 S_{zh}^2 R_{2h}^2 + 2\mu_{4h} R_{2h} S_{yzh} - 2\mu_{4h} \lambda_{4h} R_{1h} R_{2h} S_{xzh}$$

$$- 4\lambda_{4h} R_{1h} S_{yxh} \Big) < 0 \qquad (3.11)$$

Expression (3.2) to (3.11) provides the condition under which the suggested ratio - cum-product estimators $\hat{\hat{Y}}_{RPS}^{US1}$ and $\hat{\hat{Y}}_{RPS}^{US2}$ have less mean squared error in comparison to other considered estimators.

4. Empirical study

To see the performance of the suggested estimators in comparison to other estimators considered in this chapter we are considered three natural population data sets. Descriptions of the populations are given below.

Population-I [**Lone, H.A (2015)**]

n_1=4	n_2=4	N_1=10	N_2=10
$\overline{Z}_1$=43.495	$\overline{Z}_2$=43.495	$\overline{X}_1$=149.775	$\overline{X}_2$=149.775
$\overline{Y}_1$=2.685	$\overline{Y}_2$=2.685	$S^2_{z_1}$=1.4156	$S^2_{z_2}$=117.0401
$S^2_{x_1}$=12.4609	$S^2_{x_2}$=12457.51	$S^2_{y_1}$=0.254	$S^2_{y_2}$=1.9961
C_{z_1}=0.188258	C_{z_2}=0.134108	C_{x_1}=0.339097	C_{x_2}=0.386018
C_{y_1}=0.296461	C_{y_2}=0.384968	S_{yx_1}=1.608	S_{yx_2}=144.8752
$S_{yz_1}=-0.056$	$S_{yz_2}=-7.0459$	S_{xz_1}=1.3838	$S_{xz_2}=-92.0238$
$\beta_{21}(x)$ =1.976816	$\beta_{22}(x)$=2.900364	$\beta_{21}(z)$=120468	$\beta_{22}(z)$=3.664316
S_{x_1}=3.53	S_{x_2}=111.6132	S_{z_1}=1.18979	S_{z_2}=10.81851
S_{y_1}=0.503984	S_{y_2}=1.412834	W_1=0.5	W_2=0.5

Population –II [Lone, H. A (2015)]

n_1=3	n_2=3	N_1=5	N_2=5
$\overline{Z}_1$=56.2	$\overline{Z}_2$=56.2	$\overline{X}_1$=274.4	$\overline{X}_2$=271
$\overline{Y}_1$=2520.7	$\overline{Y}_2$=2520.7	$S^2_{z_1}$=0.56	$S^2_{z_2}$=23.44
$S^2_{x_1}$=5605.84	$S^2_{x_2}$=4401.76	$S^2_{y_1}$=379360.2	$S^2_{y_2}$=115860.2
C_{z_1}=0.014447	C_{z_2}=0.079893	C_{x_1}=0.349217	C_{x_2}=0.198759
C_{y_1}=0.319827	C_{y_2}=0.109251	S_{yx_1}=39360.68	S_{yx_2}=22356.52
S_{yz_1} = 411.16	S_{yz_2} =1536.24	S_{xz_1}=38.08	S_{xz_2} = 287.92
$\beta_{21}(x)$=1.885054	$\beta_{22}(x)$=2.321147	$\beta_{21}(z)$=1.846939	$\beta_{22}(z)$=1.499033
S_{x_1}=74.87216	S_{x_2}=66.34576	S_{z_1}=0.748331	S_{z_2}=4.841487
S_{y_1}=615.9222	S_{y_2}=340.3825	W_1=0.5	W_2=0.5

Population –III [Lone, H. A. (2015)]

n_1=4	n_2=4	N_1=10	N_2=10
$\bar{Z}_1$=1832.975	$\bar{Z}_2$=1832.975	$\bar{X}_1$=116.9	$\bar{X}_2$=116.9
$\bar{Y}_1$=126.15	$\bar{Y}_2$=126.15	$S^2_{z_1}$=10439.63	$S^2_{z_2}$=10663.53
$S^2_{x_1}$=37.16	$S^2_{x_2}$=43.2	$S^2_{y_1}$=181.41	$S^2_{y_2}$=158.84
C_{z_1}=0.062684	C_{z_2}=0.05072	C_{x_1}=0.042688	C_{x_2}=0.072227
C_{y_1}=0.089972	C_{y_2}=0.122838	S_{yx_1}=18.44	S_{yx_2}=23.3
$S_{xz_1} = -239.252$	$S_{xz_2} = -240.45$	$S_{yz_1} = -1072.8$	$S_{yz_2} = -655.256$
$\beta_{21}(x)$=1.577361	$\beta_{22}(x)$=2.828575	$\beta_{21}(z)$=2.972256	$\beta_{22}(z)$=2.378116
S_{x_1}=6.0959	S_{x_2}=6.572671	S_{z_1}=102.1745	S_{z_2}=103.2644
S_{y_1}=13.46885	S_{y_2}=12.60317	W_1=0.5	W_2=0.5

Table I: Percent relative Efficiency of $\bar{y}_{st}$, t_1, t_2, t_7, t_8, t_9, t_{10}, $\hat{\bar{Y}}^{US1}_{RPS}$, and $\hat{\bar{Y}}^{US2}_{RPS}$ with respect to $\bar{y}_{st}$

Estimators	Population –I	Population -II	Population –III
$\bar{y}_{st}$	**100**	**100**	**100**
t_1	**223.75**	**239.89**	**98.98**
t_2	**19.54**	**19.99**	**64.94**
t_7	**223.85**	**240.20**	**98.99**
t_8	**19.56**	**20.00**	**64.03**
t_9	**255.02**	**261.62**	**104.51**
t_{10}	**20.68**	**20.65**	**73.74**
$\hat{\bar{Y}}^{US1}_{RPS}$	**288.39**	**308.94**	**122.80**
$\hat{\bar{Y}}^{US2}_{RPS}$	**256.49**	**329.55**	**141.13**

Conclusion:

Table 1 shows that the suggested estimators $\hat{\bar{Y}}_{RPS}^{US1}$, and $\hat{\bar{Y}}_{RPS}^{US2}$ have higher percent relative efficiencies as compared to $\bar{y}_{st}$, t_1 , t_2 , t_7, t_8, t_9 and t_{10}. Section 3 deals with the theoretical efficiency comparisons of the considered estimators. This section provides the conditions under which suggested estimators have less mean squared error in comparison to unbiased estimator $\bar{y}_{st}$, conventional separate ratio and product estimators $t_i\ (i = 1,\ 2)$ and Tailor and Lone (2014) separate ratio and product type estimators $t_i\ (i = 7,\ 8,\ 9,\ 10)$. It has been observed from the table I that the suggested estimators have highest percent relative efficiencies in comparisons to other considered estimators. Thus suggested estimators are recommended for use in practice for estimating the finite population mean when conditions obtained in section 3 are satisfied.

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MATHEMATICAL INVENTIONS: PREHISTORIC MATHEMATICS TO MATHEMATICS IN TODAY

Megha Abhiman Bhamare

Department of Mathematics,

K. V. N. Naik College, Nasik, Maharashtra, India

Corresponding author E-mail: drbhamaremegha@gmail.com

Abstract:

Mathematics is an important subject in both science and everyday life. Math is a useful tool for understanding the world and for developing mental discipline. Logical thinking, critical thinking, creative thinking, abstract or spatial thinking, problem-solving skills, and even successful communication skills are all encouraged by Mathematics. Mathematical Inventions: Prehistoric Mathematics to Mathematics in Today describes Prehistoric Mathematics and Ancient Indian Mathematics. Also, describes the Mathematics in 17th Century, Mathematics in 18th Century, Mathematics in 19th Century, Mathematics in 20th Century and Mathematics in Today. The main aim of the Chapter is to motivate the researchers and students for thinking about the inventions in Mathematics & to develop simplified techniques on top of the existing techniques.

Keywords: Mathematical Inventions, Egyptian, Babylonian, Ancient, Today's Mathematics, Laws, Theorem**.**

Introduction:

Education can be seen of as a product or a process, and it can be viewed in both a broad and technical meaning. In its broadest definition, education refers to any act or event that has a formative effect on an individual's mind, character, or physical aptitude, according to George F. Kneller, a philosopher of education. In a technical sense, education is the process by which society transfers its cultural heritage its collected knowledge, values, and skills—from one generation to another through schools, colleges, universities, and other institutions. [1]

Mathematics is an abstract study of subjects such as quantity, structure, space, and change. Mathematicians solve the truth of guessing through mathematical proofs. If mathematical structure is a good model of a real phenomenon, mathematical reasoning can provide insights or predictions about nature. The history of mathematics deals with mathematical discoveries, the emergence of mathematical methods and past notations. Before the global spread of modernity and knowledge. [2]

The field of study known as the history of mathematics is primarily a study of the origin of new discoveries in mathematics and to a lesser extent a study of standard mathematical methods and notation of the past. Before modern times and with the worldwide spread of knowledge, written examples of new mathematical developments have only come to light in a few places.

1. Prehistoric Mathematics

The oldest mathematical texts available are Plimpton 322 (Babylonian mathematics ca. 1900 BC), the Moscow Mathematical Papyrus (Egyptian mathematics ca. 1850 BC), the Rhind Mathematical Papyrus (Egyptian mathematics ca. 1650 BC) and the Shulba Sutras (Indian mathematics). All of these texts concern the so-called Pythagorean theorem, which appears to be the oldest and most widespread mathematical development after arithmetic and geometry. Egyptian and Babylonian mathematics were then developed further into Greek and Hellenistic mathematics, which is generally regarded as one of the most important in greatly expanding both the method and the scope of mathematics. The mathematics developed in these ancient civilizations was then further developed and greatly expanded in Islamic mathematics. Many Greek and Arabic texts on mathematics were then translated into Latin in medieval Europe and developed further there. [3] A striking feature of the history of ancient and medieval mathematics is that bursts of mathematical development were often followed by centuries of stagnation. Beginning in Renaissance Italy in the 16th century, new mathematical developments combined with new scientific discoveries were made at an ever-increasing pace, and this continues to this day. [4]

2. Ancient Indian Mathematics

Vedic mathematics began in the early Iron Age with the Shatapatha Brahmana), which approximates the value of to 2 decimal places, and the Sulba Sutras (c. 800-500 B.C.) were geometry texts that used irrational numbers, prime numbers, the rule of three, and cube roots; calculates the square root from 2 to 5 decimal places; gave the method of squaring the circle; solved linear and quadratic equations; developed Pythagorean triples algebraically and gave a statement and a numerical proof of the Pythagorean theorem.[5] Panini (ca. 5th century BC) formulated the rules of grammar for Sanskrit. Between 400 B.C. 200 BC and 200 AD Jain mathematicians began to study mathematics for the sole purpose of mathematics. They were the first to develop transfinite numbers, set theory, logarithms, fundamental laws of indices, cubic equations, quadratic equations, sequences and progressions, permutations and combinations, squaring and taking square roots, and finite and infinite powers.[6] The Bakhshali manuscript, written between 400 BC and 200 AD, contained solutions of linear equations with up to five unknowns, the solution of the quadratic equation, arithmetic and geometric progressions, composite series, quadratic indefinite equations, simultaneous equations, and the use of zero and negative numbers. Accurate calculations for irrational numbers could be found, including calculating square roots of numbers up to a million to at least 11 decimal places. [7]

3. Mathematics in 17th Century

The 17th century saw an unprecedented explosion of mathematical and scientific ideas across Europe. Galileo, an Italian, observed the moons of Jupiter orbiting this planet with a telescope based on a toy imported from Holland. Tycho Brahe, a Dane, had collected an enormous amount of mathematical data describing the positions of the planets in the sky. His student Johannes Kepler, a German, began working with this data. Partly because he wanted to help Kepler with his calculations, John Napier in Scotland was the first to study natural logarithms. Kepler succeeded in formulating mathematical laws of planetary motion. Analytical geometry developed by Ren Descartes (1596-1650), a French mathematician and philosopher, made it possible to represent these orbits in Cartesian coordinates on a graph. Building on the earlier work of many mathematicians, Isaac Newton, an Englishman, discovered the laws of physics, explained Kepler's laws, and brought together the concepts now known as calculus. Independently of this, Gottfried Wilhelm Leibniz developed calculus and a large part of the calculus notation still used today in Germany. Science and mathematics had become an international endeavor that would soon spread across the world. [8, 9]

In addition to the application of mathematics to the study of the heavens, applied mathematics began to expand into new realms with the correspondence of Pierre de Fermat and Blaise Pascal. In their discussions of a game of chance, Pascal and Fermat laid the foundations for the study of probability theory and the corresponding rules of combinatorics. With his wager, Pascal attempted to use newly developed probability theory to argue for a life devoted to religion, on the grounds that even when the probability of success was small, the rewards were infinite. In a way, this heralded the development of utility theory in the 18th-19th centuries. [10, 11]

4. Mathematics in 18th Century

The most influential mathematician of the 18th century was probably Leonhard Euler. His contributions range from justifying the study of graph theory with the problem of the seven bridges of Königsberg to the standardization of many modern mathematical terms and notations. For example, he named the square root of minus 1 the symbol i, and he popularized the use of the Greek letter, which represents the ratio of a circle's circumference to its diameter. He made numerous contributions to the study of topology, graph theory, analysis, combinatorics and complex analysis, as evidenced by the large number of theorems and notations named after him. [12]

Other important European mathematicians of the 18th century were Joseph Louis Lagrange, who pioneered number theory, algebra, differential calculus, and the calculus of variations, and Laplace, who, in the Napoleonic era, did important work on the foundations of celestial mechanics and statistics. [13, 14]

5. Mathematics in 19th Century

During the 19th century, mathematics became more and more abstract. Carl Friedrich Gauss (1777-1855) lived in the 19th century. In addition to his many contributions to science, in pure mathematics he did revolutionary work on functions of complex variables, in geometry, and on the convergence of series. He provided the first satisfactory proofs of the fundamental algebra theorem and the quadratic law of reciprocity. [15]

This century saw the development of the two forms of non-Euclidean geometry, where the parallel postulate of Euclidean geometry no longer holds. Russian mathematician Nikolai Ivanovich Lobachevsky and his rival, Hungarian mathematician Janos Bolyai independently defined and studied hyperbolic geometry, where uniqueness of parallels no longer holds. With this geometry, the sum of the angles in a triangle is less than 180. [16]

Elliptic geometry was later developed by the German mathematician Bernhard Riemann in the 19th century; no parallel can be found here and the angles in a triangle add up to more than 180. Riemann also developed Riemannian geometry, which unifies and largely generalizes the three types of geometry, and he defined the concept of a manifold containing the ideas of curves and surfaces generalized. The 19th century saw the beginning of a great deal of abstract algebra. William Rowan Hamilton in Ireland developed non-commutative algebra. The British mathematician George Boole developed an algebra that soon evolved into what is known as Boolean algebra, in which the only numbers were 0 and 1 and in which, as is well known, 1 + 1 = 1. Boolean algebra is the starting point of mathematical logic and has important applications in computer science. Augustin-Louis Cauchy, Bernhard Riemann and Karl Weierstrass reformulated the calculus more strictly. [17, 18]

In addition, the limits of mathematics were explored for the first time. Niels Henrik Abel, a Norwegian, and variste Galois, a Frenchman, proved that there is no general algebraic method to solve polynomial equations of degree greater than four. Other 19th-century mathematicians used this in their proofs that ruler and compass alone are not sufficient to trisect any angle, to construct the side of a cube twice the volume of a given cube, nor to construct a square, which has the same area as a given circle. Since the time of the ancient Greeks, mathematicians had tried in vain to solve all these problems. [19]

Abel and Galoiss' investigations into the solution of various polynomial equations laid the foundation for the further development of group theory and the associated areas of abstract algebra. In the 20th century, physicists and other scientists saw group theory as the ideal way to study symmetry. In the late 19th century, Georg Cantor laid the first foundations of set theory, which allowed for a rigorous treatment of the notion of infinity and became the common language of almost all mathematics. Cantor's set theory and the rise of mathematical logic in the hands of Peano, L.E.J. Brouwer, David Hilbert, Bertrand Russell, and A.N. Whitehead, initiated a long-running debate about the foundations of mathematics. [20]

Several national mathematical societies were founded in the 19th century, the London Mathematical Society in 1865, the Socit Mathmatique de France in 1872, the Circolo Mathematico di Palermo in 1884, the Edinburgh Mathematical Society in 1883, and the American Mathematical Society in 1888. [21]

6. Mathematics in 20th Century

In the 20th century, mathematics became a main occupation. Thousands of new Ph.D. degrees in mathematics are awarded each year, and positions are available both in the classroom and in industry. In earlier centuries, there were few creative mathematicians in the world at any one time. Most mathematicians were either born to wealth, like Napier, or supported by wealthy patrons, like Gauss. A few, like Fourier, made a meager living teaching at universities. Unable to get a job, Niels Henrik Abel died in poverty at the age of 26 from malnutrition and tuberculosis. [22, 23]

In a 1900 speech to the International Congress of Mathematicians, David Hilbert presented a list of 23 unsolved problems in mathematics. These problems, which span many areas of mathematics, formed a central focus for much of 20th-century mathematics. Today 10 are solved, 7 partially solved and 2 still open. The remaining 4 are too loosely worded to be called solved or not. Famous historical conjectures have finally been proven. In 1976, Wolfgang Haken and Kenneth Appel used a computer to prove the four-color theorem. Building on the work of others, Andrew Wiles proved Fermat's Last Theorem in 1995. Paul Cohen and Kurt Gdel proved that the continuum hypothesis is independent (neither proved nor disproved) of the standard axioms of set theory. [24]

Mathematical collaborations of never-before-seen extent and scope occurred. The classification of finite simple group("enormous theorem"), whose proof took 500-plus journal articles by around 100 authors and tens of thousands of pages between 1955 and1983. Under the pseudonym "Nicolas Bourbaki" a group of French mathematicians, notably Jean Dieudonne and Andre Weil, aimed to present all known mathematics as a coherent, rigorous whole. The dozens of volumes that resulted have had a contentious impact on mathematics education. [25]

In 1929 and 1930, it was demonstrated that the truth or falsity of all statements made about natural numbers plus one of addition and multiplication could be determined by algorithm. Kurt Gödel discovered in 1931 that this was not the case for natural numbers including both addition and multiplication. Peano arithmetic suffices for much number theory, including the concept of prime numbers. Because of Godel's two incompleteness theorems, truth always outruns evidence in every Mathematical system containing Peano arithmetic (including all of analysis and geometry). As a result, mathematics cannot be reduced to its simplest form. [26]

Self-educated Srinivasa Aiyangar Ramanujan (1887–1920) proved over 3000 theorems, including properties of highly composite numbers, the partition function and related asymptotics, and mock theta functions. Gamma functions, modular forms, divergent series, hyper geometric series, and prime number theory were among the topics he researched. [27]

7. Mathematics in Today

The most notable modification within the field of arithmetic within the late twentieth and early twenty first centuries has been the growing recognition and acceptance of probabilistic strategies in several branches of the topic, going well on the far side their ancient uses in mathematical physics. In 2000, the clay Mathematics institute declared the seven millennium prize issues, and in 2003. [28]The Poincare conjecture was resolved by Grigori. Ben Green and Afer Tao in 2004 proved that the set of prime numbers includes long arithmetic progression. In 2006 Tao being awarded as a Fields honor. [29]

In our lives and in the progress of our mathematics, the computer also plays a completely positive function. The computer is revolutionizing mathematics by bringing certain issues to the forefront; it is even inspiring mathematicians to develop new fields of study such as computational complexity theory, automata theory, and mathematical cryptology. Simultaneously, it relieves us of some of the most boring components of traditional mathematical activity, which it performs faster and more correctly than we can. It allows us to perform numerical work quickly and comfortably, allowing us to combine our analysis of a problem with the actual calculation of numerical examples. On the other hand, the computer makes several mathematical procedures that were popular in the past outdated. As demonstrated by the discoveries of the last half of the 20th century, mathematics can enrich not only physics and other scientific disciplines, but also medicine and biomedical sciences and engineering. It can also play a role in matters as practical as speeding up the flow of Internet traffic or sharpening the transmission of digitized images, better understanding and potentially predicting stock market patterns, and even enriching the entertainment world with contributions to digital technology. Through mathematical modeling, numerical experiments, analytical studies, and other mathematical techniques, mathematics can make enormous contributions in many areas. Mathematics has to do with human genes, the world of finance and geometric movements. For example, science now has a vast amount of genetic information, and researchers need mathematical methods and algorithms to sift through the data, as well as clustering methods and computer models to interpret the data. Finance is very mathematical; it has to do with derivatives, risk management, portfolio management and stock options. All of this is modeled mathematically, and consequently mathematicians have a real impact on how these companies perform. Motion driven by the geometry of interfaces is ubiquitous in many areas of science, from growing crystals for semiconductor fabrication to tracking tumors in biomedical images.

Conclusion:

It is concluded that, there are numerous compelling reasons to study mathematical inventions. It allows students to have a better understanding of the mathematics they have already learned by demonstrating how it has evolved over time and in different locations. It promotes creative and flexible thinking by letting pupils to see historical evidence that several and entirely valid methods of viewing things and performing computations exist. Seeing the existing mathematical concepts, they will have generated a thought process to invent newer theorems and algorithms for further research in Mathematics. They would also be inspired to create simpler techniques on top of the existing ones.

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STATISTICAL INFERENCE

R. R. Tembhurne

Department of Botany,

Sangola College, Sangola, Dist.-Solapur

Corresponding author E-mail: ramesh_tembhurne@rediffmail.com

There are several different tests that we can use to analyze data and test hypothesis. This type of test that we choose depends on the data available and what question we are trying to answer. We analyze simple descriptive statistics such as the mean, median, mode and standard deviation to give us an idea of the distribution and to remove outliers, if necessary. We calculate probabilities to determine the likelihood of something happening. Finally we use regression analysis to examine the relationship between two or more continuous variables. In this chapter we are studying the chi square test and students't' test.

A) Chi Square Test:

The primary distribution between a chi square test and the tests we have worked with before is that chi square test are for used for categorical data. The chi square test can be used to estimate how closely the distribution of a categorical variable matches an expected distribution (the goodness fit test), or to estimate whether two categorical variables are independent of one another (the test of independence). The chi square test of independence is a natural extension of what we did earlier with contingency tables to examine whether or not two variables appeared to be independent of each other. In this chapter we will examine the goodness of fit test in more data.

The Greek letter 'chi' written as χ, is the symbol used to identify a chi square statistics which we will use here to evaluate how well a set of observed categorical data fits a hypothesized distribution. The chi square statistics is actually pretty straight forward to calculate.

$$\chi^2 = \frac{\sum(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

Observed = actual count values in each category

Expected = the predicted (expected) counts in each category if the null hypothesis were true.

Chi square is applied in biostatistics to test the goodness of fit to verify the distribution of observed data with assumed theoretical distribution. Therefore it is measure to study the difference of actual and expected frequencies. It has great use in biostatistics especially in sampling studies.

In Sampling studies we never expect that there will be perfect coincidence between expected and observed frequencies. Since chi square the difference between actual and expected frequencies, χ^2 is zero. Thus the chi square test describes the discrepancy between theory and observation.

Characteristics of χ^2 Test:

1. The test is based on events or frequencies and based on mean or standard deviation etc.
2. The test can be used between the entire set of observed and expected frequencies.
3. To draw inferences this test is applied especially testing the hypothesis.
4. It is general test and highly useful in research.

Assumption:

1. The observation must be large.
2. All the observation must be independent.
3. All the event must be mutually exclusive.
4. For comparison purposes the data must be in original units.

Degree of Freedom:

When we compare value of χ^2 with the table value the degree of freedom is evident. The degree of freedom means the number of classes to which values can be assigned. If we have n number of observed frequencies, the corresponding χ^2 distribution will have (n-1) degree of freedom. For example in the case of tossing of coins there are also two possibilities or classes namely head and tail. Here d*f* = n – 1 i.e. n = head and tail. Therefore d*f* = 2- 1 =1. In such away that, according to classes we fix d*f* namely n – 1.

How to calculate chi square value:

1. A hypothesis is established i.e. null hypothesis.
2. Calculate the difference between observed and expected value (O -E).
3. Square the difference $(O – E)^2$.
4. Divide the difference by its expected frequency $(O - E)^2/E$.
5. Add the obtained values in formula $\sum (O – E)^2/E$. 6. Find the χ^2 from table at certain level of significance usually 5 % or 1 % level.

Inference:

If the calculated value of χ^2 is greater than the table value of χ^2 at certain degree of level of significance we reject the hypothesis. If the calculated value of χ^2 is zero. The observed value and expected values completely coincide. If the calculated value of χ^2 is less than the table value the certain degree of level of significance, it is said to be non significant. It implies that the difference between the observed and expected frequencies may be due to fluctuations in sampling.

On the basis of chi square test some examples are solved as below.

Example 1: A cross involving different genes gave rise to F_2 generation of tall and dwarf in the ratio of 110:90. Test by means of chi square whether this value is deviated from the Mendel's monohybrid ratio 3:1.

Solution: Steps:

1. Null hypothesis:

1. There is no difference between 110:90 and Mendel's monohybrid ratio 3:1.
2. Level of significance is 5%.
3. Determining the expected values (E).
4. Mendel's monohybrid ratio Tall : Dwarf = 3:1.
5. Observed total number =110 + 90 = 200.
6. Expected values = Tall : Dwarf = 3:1 i.e. 150:50 =200.
7. Fixing of degree freedom d*f* = n – 1 = 2-1=1.
8. Result: Calculated χ^2 value is 42.6.

For 1 d*f*, at 5% level of significance the table value = 3.84

9. Inference: The calculate χ^2 value is 42.6 is greater than the table value 3.84. Therefore the hypothesis is rejected. In other words the value 110:90 is deviated from Mendel's monohybrid ratio.

Calculation:

$\chi^2 = \sum (O - E)^2 / E$

Where O = observed value and E = Expected value

Variables	O	E	O-E	$(O-E)^2$	$(O-E)^2/E$
Tall	110	150	-40	1600	10.6
Dwarf	90	50	40	1600	32.0
					42.6

$\chi^2 = \sum (O - E)^2 / E = 42.6$

Example 2: When two heterozygous pea plants are crossed, 1600 plants are produced in F_2 generation out of which 940 are yellow round, 260 are wrinkled, 340 are green round and 60 are green wrinkled. By means of chi square test whether these values are derived from Mendel's dihybrid ratio 9:3:3:1.

Solution: Steps:

1. Null hypothesis:

1. There is no difference between 110:90 and Mendel's monohybrid ratio 9:3:3:1.
2. Level of significance is 5%.
3. Determining the expected values (E) related to the dihybrid ratio 9:3:3:1.

Yellow Round = 9 Total 1600 $\therefore$ E = 9/16 x 1600 = 900

Yellwo Wrinkled = 3 Total 1600 $\therefore$ E = 3/16 x 1600 = 300

Green Round = 3 Total 1600 $\therefore$ E = 3/16 x 1600 = 300

Green Wrinkled = 1 Total 1600 $\therefore$ E = 1/16 x 1600 = 100

4. Observed Values: Yellow Round : Yellow Wrinkled : Green Round : Green Wrinkled = 940:260:340:60
5. Fixing of degree freedom d*f* = n-1=4-1=3

Calculation:

$\chi^2 = \sum (O - E)^2 / E$

Where O = observed value and E = Expected value

Variables	**O**	**E**	**O-E**	**$(O-E)^2$**	**$(O-E)^2/E$**
Yellow Round	940	900	40	1600	1.77
Yellow Wrinkled	260	300	-40	1600	5.33
Green Round	340	300	40	1600	5.33
Green Wrinkled	60	100	-40	1600	16
					∑27.43

$\chi^2 = \sum (O - E)^2 / E = 27.43$

6. Result: Calculated 2 value is 2743

For d*f*, at 5 % level of significance the table value = 7.81

7. Inference: The calculated χ^2value is 27.43 is greater than the table value 7.81. Therefore the hypothesis is rejected. In other words there is no real independent assortment i.e the observed values are deviated from Mendel's dihybrid ratio.

Significance:

It is used to test the goodness of fit. The test enables to find out whether the difference between the expected and observed values is significant or not. If the difference is little then the fit is good otherwise fit is poor.

Introduction:

There are several statistical tests that use the t-test distribution and can be called a t-test. One is student’s t-test for one sample, named after student the pseudonym that William Gosset used to hide his employment by the Guinness brewery in the early 1950s (they had a rule that their employees weren’t allowed to publish, and Guinness didn’t want other employees to know that they were making an exception for Gosset). Students t-test for one sample compares a sample to a theoretical mean. It has so few uses in biology. When you have one measurement variable, and you want to campare the mean value of the measurement variable to some theoretical expectation then you use students t-test. If the sample size is less than 30 i.e. $n<30$ then those sample may be regarded as small samples. Principles of statistical inference are the same as in large sample but the techniques differ in the case of small samples. Here student t-test can be used. It is commonly used in fields such as physics and product testing in drug science. It is rare to have this kind of theoretical expectation in biology so you will probably never use the one sample t-test.

A t-test is most commonly applied when the test statistic would follow a normal distribution. If the value of a scaling term in the test statistic were known. When the scaling term is unknown and is replaced by an estimate based on the data, the test statistics (under certain condition) follow a students t distribution. The t-test can be used for example to determine if two sets of data are significantly different from each other.

t-test are normally used to campare the means of two samples of numeric data to determine whether they are significantly different from one another, although there is such a thing as a one sample t-test, which has a related but slightly different purpose. The data can be either be continuously distributed or discrete as long as they have a normal distribution.

Continuously distributed numeric variables are one that in principle, can take an infinite number of values if measured precisely enough-for example: body mass, height, nitrogen concentration in a water sample, or cholesterol level in the blood stream.

Descrete numeric variables are ones that can take only a certain set of values-for example: the number of leaves on a tree, the number of bacterial colonies on a petridish. Both these variables can take only integer values, although the number of possible values is very large.

To learn how to determine whether data have a normal distribution. If data are not normally distributed, alternative tests are available.

If there are three or more samples of data, rather than just one or two then one factor analysis of variance may be used.

For testing hypothesis about means when your data have a normal distribution there are three types of t-test.

A one sample t-test is used to compare the mean of a single sample with a value that is expected bases on some prior knowledge. For example the long term average high temperature in the Twin Cities on 1 April is 50^{o} F. If we had data on the high temperature for each 1 April from 1998-2017 (n=20), and if these data had a normal distribution we could perform a one sample t-test to determine whether the average high for the past 20 years was significantly different from the long term average 50^{o} F.

A two sample t-test is used to compare two sample means to determine if they are significantly different from each other. For example if we had data on the Twin Cities 1 April high temperature for the years 1978-1997 and 1998-1017, we could use a two sample t-test to determine whether the average high for the most recent 20 year period was significantly different from the average high for the previous 20 year period.

A period t-test us used to compare two sample means if each value within one of the samples can be sensibly paired with an equivalent value in the other sample. For example if we had data on the 1 April high temperature in Duluth for 1998-2017, we could do a paired t-test to determine whether the average Duluth temperature during this period was significantly different from the average Twin Cities temperature during the same period. In this example, the two high temperature for 1998 (Duluth and Twin Cities) can be sensibly paired with one another as can every other pair of temperature taken on 1 April of the same year. Contrast this with the two sample test above where there is no sensible justification for pairing 1978 in the first sample with 1998 (or any other year) in the second sample.

The null hypothesis for the independent samples t-test is μ_1-μ_2. In other words, it assumes the means are equal. With the paired t-test the null hypothesis is that the pair wise difference between the two test is equal (H_O: μ_d-0). The difference between the two tests is very subtle which one you choose is based on data collection method.

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TEACHING-LEARNING OF LIMITS OF FUNCTIONS: A TEACHER-STUDENT PERSPECTIVE

Kailas S. Borase

Department of Mathematics,

RNC Arts, JDB Commerce and NSC Science College, Nasik Road-422 101

Corresponding author E-mail: ksb.ganit@rediffmail.com

Abstract:

The research can be taken up this experimental research with a view to assess an effectiveness of teaching-learning process in the concept of limits of functions in mathematics through the graphical approach in raising the overall the knowledge, skill, and attitude towards the mathematics. The graphical approach in understanding the mathematical concept of limits of functions through the Inquiry Oriented Approach (IOA)[3] model can have found effective in enhancing the level of understanding of this concept.

1. Introduction:

Together with philosophy, Mathematics is the oldest academic discipline known to human being[3]. Currently, mathematics is a huge complex enterprise, far beyond the keen of anyone individual. Those of us who choose to study the subject can only choose a micro of it, and in the end must specialize rather drastically in order to make any contribution to the evolution of ideas involved.

This research article in this chapter provides an outline of the current research in students' understanding of topics in limits of functions. The intent of work is to provide an overview of specific difficulties based on education research in the subject communication in Mathematics in the context of limits of functions.

1.1. Limits of functions

The limits of the functions, in brief limits concept is an important part of the foundations of mathematical analysis and not understanding it clearly might lead to problems when dealing with concepts such as infinity, infinitesimals, convergence, continuity and derivatives which are the main aspects of Calculus. If the student grasps the concept of limits, the above connected concepts become easier to work with, but it is difficult for the students to make sense of this concept. In India, limits of functions are the main topic treated almost in all branches viz. graduate and undergraduate as well as Engineering, Science, Commerce, and Chartered

Accountants etc. Here attempts have been made the approach of students for understanding mathematics through geometrical one rather the conventional one for the limits.

In fact, even many great mathematics researchers have found it hard to accurately handle limits through time, this is one more.

1.2 Importance and learning of mathematics

Learning Mathematics is the endeavor requiring a number of abilities, which may vary from different mathematical topics. How students learn mathematics, may also vary. Mathematics has several characteristic properties viz. the use of models describing the real world, the compact and unambiguous formulations for clear expositions, and the deductive reasoning in proving problem solving. Each property offers its own set of challenges for mathematics students as well as teachers, for example, from a model to the real world, from everybody language to a mathematical expression or from one step to another in mathematical proof.

As mathematics is the queen of science, calculus is the soul of mathematics and limits is at the heart of the calculus. Hence, proper learning of mathematics is the most important part of mathematics in teaching-learning process of mathematics. One aspect focused on this research article is transition, e.g. in inquisition and replication.

1.3 History of mathematics in context of limits of functions

Mathematical history is exciting, and it is a significant slice of the intellectual pie. A good education consists of learning difficulties of the students at different methods of conversation, and certainly, mathematics is one of the most well developed and important models of conversation that the world has observed so far.

For many centuries, the idea of a limit was confused with vague and something philosophical ideas of infinity i.e. infinitely large, infinitely small numbers and other mathematical entities. The idea of limit was also confused with subjective and undefined geometric intuitions. Here in this research article, the researcher’s aim is to highlight the contribution of geometrical approach in understanding the concept of limits.

The history of limits of functions shows that it was not obvious how a definition of limits should be stated or even if limits were useful. One of the research in which the idea of limit was introduced to resolve three types of difficulties[1]:

- Geometric problems, e.g. the calculations of area, ‘exhaustion’ and consideration of the nature of geometric lengths;
- The problem of the sum and rate of convergence of a series;

- The problems of differentiation that come from the relationship between two quantities that simultaneously tends to zero.

This research work focuses on calculus teacher's knowledge of student thinking about limit and, using historical development as a lens that explores the nature of the difficulties associated with the concept of limit. More specifically, the research study addresses the following questions:

(a) What do the teachers of calculus know of their student's thinking of limit? and

(b) How can the historical development of the limit help us make sense of college teacher's knowledge of student thinking about limit?

2. Purpose and significance of the study:

The researcher became aware of student's problems with the idea of the concept of limits in a huge branch of Calculus in vigorous subject Mathematics. The concept of limit is included in the curriculum of XII^{th} Science and undergraduate courses of Science stream at University level of an Indian education system. This covers the definition of a limit, formal as well as epsilon-delta form, a limit as x approaches to infinity and the obvious relation between limits and continuity in addition to these properties of the limits, continuity, problems on continuity. To finish the same, the little time, about four clock hours is given for its teaching-learning in University curriculum.

Research in common parlance refers to search for knowledge. One can also define as a scientific and systematic search for pertinent information on a specific topic. Indeed, research is an art of scientific investigation. The advance learner's dictionary of current English lays down meaning of research as a careful investigation or inquiry especially through search for new facts in any branch of knowledge. "All progress is born of inquiry. Doubt is often better than confidence, which leads to inquiry, and inquiry leads to invention and invention finally can leads to replication." The said quotation is an extension of famous Hudson Maxim in context of significance of research.

There are three ways to study Mathematics through the following three approaches:

1) Analytical Approach,
2) Geometrical Approach,
3) Practical Approach.

Here the researcher used to apply second approach of geometrical to better understand the concept of limits instead of the traditional way. Geometrical approach is nothing but the graphical way of understanding the concept in mathematics. As far as the concept of limit is concerned, it is defined on the functions and functions can be drawn by using graphs at all times and once able to draw the graphs, apply the concept of understanding of limit via graphical approach, then you are through. Many attempts have been made to break through this dilemma but without much success.

Hence, there is an urgent need to develop some Inquiry Oriented Approach (IOA) in Learning Difficulties of Mathematics in general and especially for limit theory. Here the researcher focuses on the theory of limits and hopes that this IOA can be applied to the rest fields of Mathematics.

3. Scope of the study:

The following limitations of the research study have been observed.

1. The study can be limited to your own class of mathematics background students.
2. The authenticity of the data used depends entirely on the accuracy of such data.
3. During course of personal interviews, the prejudices or bias on part of interviews may have influence on the response received.
4. The study comprises both genders in equal numbers to overcome the gender differences.
5. Time is the biggest constraint.
6. Sample is randomly selected.
7. Questionnaire tool is used for data collection.
8. Replication Principle is used for selected students.

4. Objectives of the study:

As the main body of mathematical analysis, Calculus is mankind's a greatest discovery in the 20th century, and a statue of human's wisdom. Limit theory is the basic theory of calculus, and limit concept is the core concept of limit theory, henceforth it is very important for students to learn limit concept well. However, to teach limit concept well is a worldwide difficult job as far as teaching learning concerned, and this situation urgently need to be changed by times and calculus teaching reform.

On the other hand, inquiry teaching has made remarkable achievements on elementary mathematics teaching reform, but there is not even a single progress on advanced mathematics teaching reform. Absolutely, the way in elementary mathematics teaching reform is not quit fit advanced mathematics teaching reform. Therefore, combining with improving students' learning style and learning enthusiasm, limit concept inquiry teaching is a very important significant research subject. Here research objective itself makes a question originated in hypothesis-born from individual's experience right from the student till to a mathematics teacher and discussions with colleagues-which "inquiry" practices in the form of definitions, properties, examples, problems, curiosity, confidence, intelligence, skill etc. strongly influence what students learn about limits at college level.

In order to progress this experimental research among junior college and undergraduate students in a class of students, it had proposed in mind the following objectives.

- To diagnose learning difficulties along with misconceptions in the concept and applications in Limit Theory (Limits of Functions).
- To raise the level of students' living, impact of literacy of their parents and curiosity and confidence in their own in about this concept during the teaching-learning process.
- To analyze gender differences and/or differences in living status and/or impact of literacy of the parents of students' in their study and/or overall curiosity, confidence of the students for the topic considered and hence in mathematics, in general.
- To prescribe and introduce the graphical approach, called the inquiry-oriented approach (IOA) of understanding the concept of limits better than the formal approach.

5. Experience of the study:

Mathematics at the higher secondary and undergraduate level in India is formally presented in textbooks and at lectures. An initial course in mathematics at Indian universities usually comprises algebra and calculus including the notion of limits of functions, which has been proven to be difficult for students to learn at a formal base only. Many different aspects regarding the notion of limits and the nature of learning that cause these difficulties are addressed in this research study. The experience, as a student and later on as a college mathematics teacher, implied to researcher that learning limits of functions expense time and effort, perhaps to a greater extent than other parts of basic calculus. Here, the expectations wanted to understand more about how students perceive and learn limits of functions. The overall research question became: How do students deal with the concept of limits of functions at the basic higher secondary and undergraduate level in India? In an attempt to answer this vast question, it is conducted studies at a large in five stages in a Mathematics class. No such study on limits probably had previously been done in India and it was therefore compared mostly to foreign research results. It is important that teachers who work with college level mathematics education are aware of the learning situation of students and are prepared to meet them at their levels in their teaching- learning processes, meaning that the pre-knowledge of students is crucial.

6. Hypothesis:

A statistical hypothesis is unproven statement about the distribution of the random variables under consideration.

H_0:- There is no significant difference in the achievement of learning 'limit theory' after using IOA in the classroom.

H_1:- There is significant difference in the achievement of learning 'limit theory' after using IOA in the classroom.

The conclusion will be drawn according to the acceptance or rejection of the hypothesis H_0.

7. Methodology adopted:

To carry out any type of research work an adoption of correct methodology is an art and way of success for that particular research. The researcher shall choose the most appropriate instruments procedures and methods that will provide the collection and analysis of data upon which hypothesis may be tested. The researcher shall a meet with mathematics teachers teaching mathematics especially Calculus at junior colleges and undergraduate classes to seek their opinion about inquisition and replication with difficulties in understanding the concept of limit in teaching-learning process of mathematics.

The investigator shall assume the research work as an experimental research for assessing the effectiveness in enhancing the level of knowledge, curiosity level along with parents' literacy in addition to the residential status of the students involved under the study. This is an experimental research for assessing the effectiveness of the strategy of adopting the geometrical approach especially graphical view in understanding of the limit concept. Much attempt may have been done so far with this, but this can be one more with specially covered the psychological attitudes among the students, together with the residential impact for their understanding in topic considered.

As an example, students from Junior college and undergraduate level with mathematics are one of the subjects were taken as population for this research study, which is summarizing in the following Table 1.1.

Table 1.1: Distribution of the sample for the study

Sex/Group	Controlled Group	Experimental Group	Total
Boys	125	125	250
Girls	125	125	250
Total	250	250	**500**

8. Design of the study:

The sample had divided into two groups namely the controlled group (CG) and an experimental group (EG) with equal number of boys and girls students from the sample.

The research work was conducted at the college. The researcher lectures mathematics in the college. The students are enrolled for the 12th Science undergraduate students with mathematics is one of the subjects. The main topics are limits, continuity, differentiation and integration including their applications in real-life. It is multi-cultural, multi-status classroom and the students are taught through the medium of English, which is their second language.

The concepts of limits under the study were to be taught in class by making use of discussions, problem solving tasks, viz. interviews and questionnaires etc. These tasks were to assessed in order to determine possible misconceptions of the limits of functions and through pre-test, post-test and retention-test by understanding the limit concept using graphical approach more better by conventional one. The same planed to analyze with the help of several statistical tools viz. t-test, chi-test, z-test ANOVA etc. Absolutely, might be the first time use of principle of replication in this research study.

It had been prepared four tests Pre-Test, Post-Test, Retention-Test and the test for effectiveness of Principle of Replication in five stages accordingly with four major related topics of the limits of functions which is depicted in the following Table 1.2

Table 1.2: Plan of research study

Test Level	Plan of Period	Actual Action Period
Concept I: Basic concept of limits of functions	At the beginning of the research	August 2010
Concept II: Epsilon-delta definition of limit	After Two Weeks	August 2010
Concept III: Problems of limit	After Four Weeks	September 2010
Concept IV: Problems of continuity	After Four Months	January 2011

9. Variables of the study:

In this typical research experiment, there are two types of variables used-independent and dependent. An independent variable is the variable that scientist manipulates (the treatment) to determine its effect on some research (the dependent).

In present study, it has taken into account eight types of variables as observed from the specific objectives and corresponding to the null hypothesis as mentioned earlier.

These variables are as under[2]:

Pedagogical Dependent Variables (Attributes)

- Curiosity gained
- Confidence gained
- Literacy status of the family
- Residential status of the family

Psychological Independent Variables

- Intelligence
- Interest
- Attitude
- Skill

10. Organization of the study:

It can be organized the plan of study as in Table 1.1 following ways, there are five stages; A, B, C, D, and E in which Pre-Test, Post-Test and Retention- Test was planned to conduct for the study in hand for both the Controlled group (CG) and the Experimental group (EG).

Table 1.3: Stages of research study

Test	Teaching-Learning and Questionnaire Schedule	Groups	Stages
Pre-Test	-	CG+EG	A
-	About limits of Functions	EG	B
Post-Test	-	CG+EG	C
Retention-Test	-	CG+EG	D
-	Proposition Type Questionnaires	Randomly Selected 100 respondents from EG	E

11. Population and sample for the study:

An experimental research cannot be done without the population. Here, the sample is a group of students, which will evaluate the applicability of an Inquiry Oriented Approach (IOA) technique.

Total Population	:	5000
Sample Size	:	500
Geographical Area	:	Local College
Sampling Procedure	:	Random Sample

The sample was taken randomly from local colleges of the city to achieve the goal of the study.

Conclusions:

This covers a research work that presents a research investigation into the effect on student conceptual understanding of the central topics in the limit concepts, and overall achievement, with incorporation of an Inquiry Oriented Approach (IOA) with the attributable variables under study. There can be several significant differences between the groups of students who completed limit concepts without the IOA. The IOA group can score significantly higher in conceptual understanding and achievement.

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k – NEAREST NEIGHBOR (k-NN) ALGORITHM FOR CLASSIFICATION

Sudha Bishnoi

Chaudhary Charan Singh Haryana Agricultural University,

Hisar, Haryana

Corresponding author E-mail: sudha.bishnoi@gmail.com

Abstract:

This chapter gives an introduction to the simplest method of machine learning which is called as k-Nearest Neighbor. The k-Nearest Neighbor algorithm (k-NN) is a supervised non-parametric machine learning model which is based on a simple distance measure, and it can be used for both classification and regression problems. The chapter starts with an introduction to the basics of machine learning and the theory of k-Nearest Neighbor with a focus on classification. In the subsequent sections, the advantages and disadvantages of k-NN will be discussed.

Keywords: k-Nearest Neighbor, supervised learning, machine learning, classification

Introduction:

Machine learning is a field in data analytics that uses statistical learning algorithms to build systems that have the capability to automatically learn and improve from experiences without being explicitly programmed (Mitchell, 1997). Machine learning algorithms are broadly categorized into two major types i.e. supervised and unsupervised learning. The supervised algorithm takes a known set of input dataset and its known outputs to learn the classification model, and this model generate an appropriate classification when a new unlabelled dataset is given. Thus, supervised machine learning aims to infer a function from labelled training data, which can be used for classifying new unknown observations. While in case of unsupervised machine learning the data is unlabelled, and the algorithms groups the unlabelled information by finding the similarities and pattern in the data.

Classification belongs to the category of supervised machine learning where the outputs are also provided with the input dataset. The learning algorithm of the classifiers is employed to build a model which is used to find the relationship between the variables and the class label of the given data. A major amount of literature within machine learning has been published on the problems of classification. In classification, the new unlabelled observations is assigned into a correct class by learning from previous labelled data. Classes are sometimes called as labels, categories or targets.

General algorithm of a machine learning classifier is illustrated in figure 1.

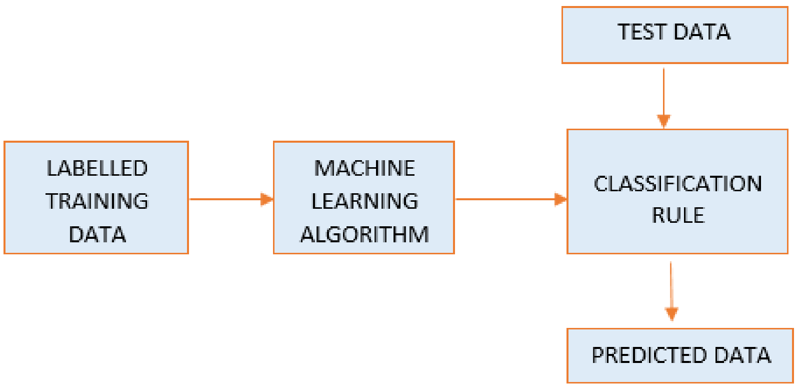


Figure 1: General structure of a classification algorithm

k-Nearest Neighbor (k-NN):

k-Nearest Neighbor is a type of supervised machine learning algorithm which can be used for both regression as well as classification problems. However, it is mainly used for classification or predictive problems. It was proposed by Cover and Hart (1967) for performing pattern classification task. The k-NN was developed with the need to perform discriminant analysis when reliable parametric estimates of probability densities are not known and are difficult to determine (Beckmann *et al.*, 2015). K-Nearest Neighbor is one of the most famous classification algorithms because it is very simple to use and ease of interpretation (Wu *et. al.*, 2008). This machine learning algorithm assumes that similar things/observations exist in close proximity. In other words, observations which are similar are close to each other. The k-NN algorithm can be defined well by the following two properties:

- **Lazy learner:** k-NN is a lazy learning algorithm as it does not explicitly learn the model, but it tries to memorize the training/labelled data. This information is then used as knowledge for the classification phase.
- **Non-parametric:** In k-NN no assumptions are made about the distribution of the underlying data. It is useful because the practical data which is available in the real world, does not obey theoretical assumptions most of the times and hence, this algorithm comes to rescue.

The k-NN Algorithm:

k-Nearest Neighbor algorithm uses similarity of variables/observations to predict the class/label of new data points, which means that the new data point will be assigned a class based on how closely it matches to the points in the training data set.

The implementation of k-NN algorithm is given as:

(i) For implementing k-NN algorithm the whole dataset is loaded, and then split into training and test dataset.

(ii) The value of k which is the number of nearest neighbor is chosen and it can be any integer.

(iii) For each observation in the dataset:

  a. Calculate the distance between the test data and each row of training data using any of the distance measures. The most widely used measure to calculate the distance is Euclidean distance.

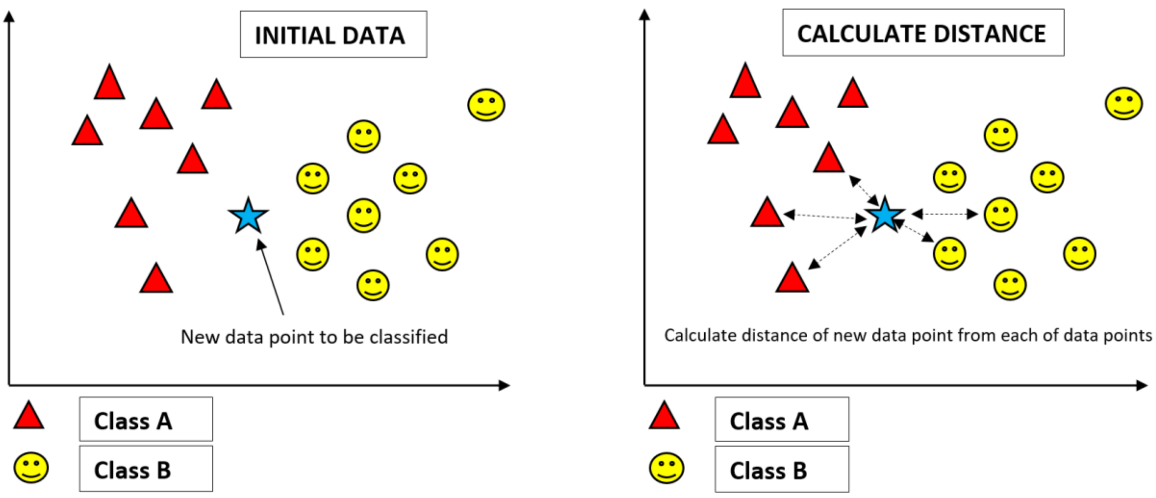
  b. Add the distance and the index of the observations into an ordered collection.

(iv) Sort the ordered collection of distances and indices in ascending order i.e. from smallest to largest by the distances.

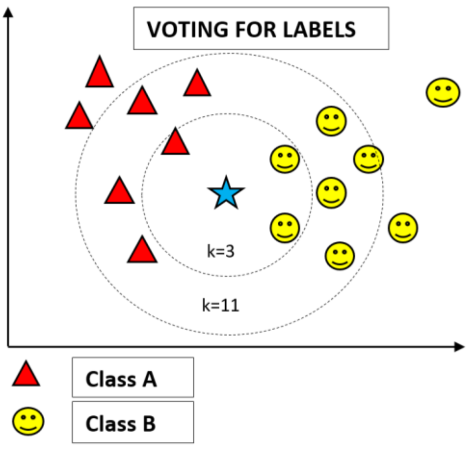
(v) Pick the first k values from the sorted collection.

(vi) Note the labels of selected k entries and assign a class/label to the test data observation based on the most frequently occurring class of these entries.

The figure 2 illustrates the working algorithm of k-Nearest Neighbor classifier.



(a) **(b)**



(c)

Figure 2: k-NN algorithm representation:

(a) new observation, (b) find distances, (c) classification based on distance

In the figure 2, suppose blue coloured star (let it as point P) is a new data point, for which label/class needs to be predicted. The k closest point/neighbor to the point P are found and then by majority voting the point P is assigned either to the class A or class B. For example, when k=3, there are two observations which belongs to the class B and one observation belongs to the class A, so by majority rule the test point P is classified to class B. Similarly, when we take k=11, majority of the nearest neighbor are again from class B, hence the new data point is assigned to the class B.

In the process of creating a k-NN classifier, k is an important parameter and different k values will cause different performances in classifying an individual. Choosing the number of nearest neighbor that is determining the value of k is the most critical problem (Mody, 2009). To select the optimum value of k, the k-NN algorithm can be run several times with different values of k and that value of k is chosen which reduces the number of errors or we can say that which increases the accuracy to make predictions when it is given the test data.

Below are some points to keep in mind while choosing the value of k:

- If the value of k is decreased to one, predictions become less stable. For example, if we take k=1 and we have a test point surrounded by several observations of class say A and one observation of class B, but that one observation of class B is the single nearest neighbor. Reasonably, we would think that the test data point most likely belong to class A, but the value of k=1 that is one nearest neighbor. So, the k-NN incorrectly classifies the test data point into the class B.

- Inversely, if the value of k is increased, the predictions become more stable due to majority voting or averaging, and hence, more likely to make more accurate classifications but up to a certain point. Eventually, we begin to witness an increase in the number of errors. It is at this point we can conclude that we have pushed the value of k too far.
- In cases where we are taking a majority vote that is picking the mode in a classification problem among class labels, we usually take k as an odd number to have a tiebreaker.
- Usually, k is taken as square root of the number of observations, and value of k can also be checked by generating the model for different values of k and checking their performance at all values.

Advantages of k-Nearest Neighbor

- The k nearest neighbor algorithm is highly unbiased in nature and there are no prior assumptions about the underlying data.
- This algorithm is mostly considered over the other classification algorithms because of its less calculation time and easy interpretation of the output.
- k-NN algorithm is easy to implement and has gained good popularity, as it is very simple and effective in nature.
- The accuracy is pretty high but not competitive in comparison to some other supervised machine learning algorithms.
- No re-training of model is required if a new training data point is added to the existing training set.
- It is a versatile algorithm and can be used for both classification and regression.

Disadvantages of k-Nearest Neighbor

- This algorithm is computationally expensive, because the algorithm need to store all of the training data.
- Classification stage gets significantly slower as the number of predictors or observations increases.
- For every unlabeled test data point, the distance has to be computed between the test data point and all the training data points. Thus a lot of time is taken for the classification phase.
- The k-NN algorithm is sensitive to irrelevant features and scale of the data.

- It is not good at classifying the boundary data points where they can be classified one way or another.
- It performs better with a lower number of features/variables and when the number of features/variables increases than it requires large data. Increase in the dimension also leads to the problem called over fitting. This problem of higher dimension is known as curse of dimensionality.

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MICRO g^*-CONTINUOUS MAPS AND MICRO g^*-IRRESOLUTE MAPS IN MICRO TOPOLOGICAL SPACES

S. Sandhiya, R. Anandhi and N. Balamani

Department of Mathematics,

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore-641043

Corresponding author E-mail: sandhiyaselvakumar29@gmail.com,

anandhi08081985@gmail.com, nbalamani77@gmail.com

Abstract:

The purpose of this chapter is to define Micro g^*-continuous maps and Micro g^*-irresolute maps in Micro topological spaces. Further we investigate the properties and characterizations of Micro g^*-continuous maps and Micro g^*-irresolute maps with pertinent examples.

Mathematical Subject Classification: 54B05, 54A10, 54C05

Keywords: Micro topological spaces, Micro continuous map, Micro g^*-continuous map, Micro g^*-irresolute map.

1. Introduction

The concept of rough set theory was studied by Pawlak [6] and he introduced the notion of lower approximation, upper approximation and boundary region of a subset of the universe. Carmel Richard [4] introduced the concept of Nano topology. The Micro topology was introduced by Sakkraiveeranan Chandrasekar [8] and he also studied the concepts of Micro pre-open and Micro semi-open sets. Further he introduced the concept of Micro continuous map. He also defined Micro pre-continuous and Micro semi-continuous maps in Micro topological spaces. Chandrasekar and Swathi [5] introduced Micro α-continuity in Micro topological spaces. Taha *et al.* [11] initiated the concept of Micro g-continuous map. Anandhi and Balamani [1,2,3] initiated the concept of Micro α-generalized closed set, separation axiom and Micro αg-continuous map in Micro topological spaces. Recently, Sandhiya and Balamani [9] introduced the concept of Micro g^*-closed sets in Micro topological spaces and analyzed some of its properties. Moreover, Micro ψ-closed sets are introduced by Sowmiya and Balamani [10]. In this chapter we have introduced a new class of Micro continuous and Micro irresolute maps called Micro g^*-continuous and Micro g^*-irresolute maps in Micro topological spaces. Also the relationship between these maps and other existing maps are obtained and their properties are analyzed.

2. Preliminaries

Definition 2.1[6] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibilty relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be in indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = U_{x\in U}\ \{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = U_{\mathrm{x\in U}}\ \{R(x): R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$

Definition 2.2[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then, $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
3. The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets and the complement of a Nano open set is called a Nano closed set.

Definition 2.3[8] Let $(U, \tau_R(X))$ be a Nano topological space. Then $\mu_R(X) = \{N \cup (N' \cap \mu): N, N' \in \tau_R(X) \text{ and } \mu \notin \tau_R(X)\}$ and $\mu_R(X)$ satisfies the following axioms:

1. U and $\phi \in \mu_R(X)$
2. The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$
3. The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$

Then, $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4[9] Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro g^*-closed if Mic-$cl(A) \subseteq L$ and L is Micro g-open in U.

Definition 2.5 Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called a

(i) Micro continuous map [8] if $f^{-1}(P)$ is Micro closed in $\left(U, \tau_R(X), \mu_R(X)\right)$ for every Micro closed set P in $\left(V, \tau_R(Y), \mu_R(Y)\right)$.

(ii) Micro pre-continuous map [8] if $f^{-1}(P)$ is Micro pre-closed in $\left(U, \tau_R(X), \mu_R(X)\right)$ for every Micro closed set P in $\left(V, \tau_R(Y), \mu_R(Y)\right)$.

(iii)Micro semi-continuous map [8] if $f^{-1}(P)$ is Micro semi-closed in $\left(U, \tau_R(X), \mu_R(X)\right)$ for every Micro closed set P in $\left(V, \tau_R(Y), \mu_R(Y)\right)$.

(iv)Micro g-continuous map [11] if $f^{-1}(P)$ is Micro g-closed in $\left(U, \tau_R(X), \mu_R(X)\right)$ for every Micro closed set P in $\left(V, \tau_R(Y), \mu_R(Y)\right)$.

(v) Micro α-continuous map [5,7] if $f^{-1}(P)$ is Micro α-closed in U for every Micro closed set P in $\left(V, \tau_R(Y), \mu_R(Y)\right)$.

(vi) Micro αg-continuous map [3] if $f^{-1}(P)$ is Micro αg-closed in $\left(U, \tau_R(X), \mu_R(X)\right)$ for every Micro closed set P in $\left(V, \tau_R(Y), \mu_R(Y)\right)$.

3. Micro g^*-Continuous Maps

In this section, Micro g^*-continuous maps in Micro topological spaces are introduced and its properties are derived. It is shown that the composition of two Micro g^*-continuous maps need not be Micro g^*-continuous.

Definition 3.1 Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called a Micro g^*-continuous map if $f^{-1}(P)$ is Micro g^*-closed in U for every Micro closed set P in V.

Example 3.2 Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{\phi, \{a\}, U\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$. Micro g^*-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c\}, U$. Let $V = \{a, b, c\}$, $V/R = \{\{a\}, \{b\}, \{c\}\}$. Let $Y = \{b, c\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, V\}$. Micro closed sets in $V$are $\phi, \{b, c\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Therefore for every Micro closed set P in V, $f^{-1}(P)$ is Micro g^*-closed in U. Hence f is Micro g^*-continuous.

Proposition 3.3 Every Micro continuous map is Micro g^*-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be Micro continuous. Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro continuous, $f^{-1}(P)$ is Micro closed. Since every Micro closed set is Micro g^*-closed, $f^{-1}(P)$ is Micro g^*-closed. Hence f is Micro g^*-continuous.

Example 3.4 Let $U = \{a,b,c,d\}$, $U/R = \{\{a\},\{b\},\{c,d\}\}$. Let $X = \{c,d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{c,d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{c,d\}, \{a,c,d\}, U\}$. Micro closed sets in U are $\phi, \{b\}, \{a,b\}, \{b,c,d\}, U$. Micro g^*-closed sets in U are $\phi, \{b\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}, U$. Let $V = \{a,b,c,d\}$, $V/R = \{\{a,b\},\{c\},\{d\}\}$. Let $Y = \{a,b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, \{a,b\}, V\}$. Micro closed sets in V are $\phi, \{c,d\}, \{b,c,d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Then f is Micro g^*-continuous but not Micro continuous, since for the Micro closed set $\{c,d\}$ in V, $f^{-1}(\{c,d\}) = \{b,d\}$ is not Micro closed in U.

Proposition 3.5 Every Micro g^*-continuous map is Micro g-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be Micro g^*-continuous. Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro g^*-continuous, $f^{-1}(P)$ is Micro g^*-closed. Since every Micro g^*-closed set is Micro g-closed, $f^{-1}(P)$ is Micro g-closed. Hence f is Micro g-continuous.

Example 3.6 Let $U = \{a,b,c,d\}$, $U/R = \{\{a\},\{c\},\{b,d\}\}$. Let $X = \{b,d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{b,d\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{b\}, \{b,d\}, U\}$. Micro g^*-closed sets in U are $\phi, \{a,c\}, \{a,b,c\}, \{a,b,d\}, U$. Micro g-closed sets in U are $\phi, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, U$. Let $V = \{a,b,c,d\}$, $V/R = \{\{a\},\{b\},\{c,d\}\}$. Let $Y = \{c,d\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{c,d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, \{c,d\}, \{a,c,d\}, V\}$. Micro closed sets in V are $\phi, \{b\}, \{a,b\}, \{b,c,d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Then f is Micro g-continuous but not Micro g^*-continuous, since for the Micro closed set $\{b,c,d\}$ in V, $f^{-1}(\{b,c,d\}) = \{b,c,d\}$ is not Micro g^*-closed in U.

Proposition 3.7 Every Micro g^*-continuous map is Micro αg-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be Micro g^*-continuous. Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro g^* -continuous, $f^{-1}(P)$ is Micro g^*-closed. Since every Micro g^*-closed set is Micro αg-closed, $f^{-1}(P)$ is Micro αg-closed. Hence f is Micro αg-continuous.

Example 3.8 Let $U = \{a,b,c,d\}$, $U/R = \{\{c\},\{d\}\{a,b\}\}$. Let $X = \{a,b\} \subseteq U$. Then, $\tau_R(X) = \{U,\phi,\{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi,\{a\},\{a,b\},U\}$. Micro g^*-closed sets in U are $\phi,\{c,d\},\{a,c,d\},\{b,c,d\},U$. Micro αg-closed sets in U are $\phi,\{b\},\{c\},\{d\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\},U$. Let $V = \{a,b,c,d\}$, $V/R = \{\{a\},\{b\},\{c,d\}\}$. Let $Y = \{c,d\} \subseteq V$. Then, $\tau_R(Y) = \{V,\phi,\{c,d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi,\{a\},\{c,d\},\{a,c,d\},V\}$. Micro closed sets in $V$are $\phi,\{b\},\{a,b\},\{b,c,d\},V$. Let $f: (U,\tau_R(X), \mu_R(X)) \to (V,\tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Then f is Micro αg-continuous but not Micro g^*-continuous, since for the Micro closed set $\{b\}$ in V, $f^{-1}(\{b\}) = \{c\}$ is not Micro g^*-closed in U.

Remark 3.9 Micro g^*-continuous maps and Micro semi continuous maps are independent as observed from the following examples.

Example 3.10 Let $U = \{a,b,c,d\}$, $U/R = \{\{c\},\{d\},\{a,b\}\}$. Let $X = \{a,b\} \subseteq U$. Then,$\tau_R(X) = \{U,\phi,\{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi,\{a\},\{a,b\},U\}$. Micro semi closed sets in U are $\phi,\{b\},\{c\},\{d\},\{b,c\},\{b,d\},\{c,d\},\{b,c,d\},U$. Micro g^*-closed sets in U are $\phi,\{c,d\},\{a,c,d\},\{b,c,d\},U$. Let $V = \{a,b,c,d\}$, $V/R = \{\{a,b\},\{c\},\{d\}\}$. Let $Y = \{a,b,c\} \subseteq V$. Then, $\tau_R(Y) = \{V,\phi,\{a,b,c\}\}$. Let $\mu = \{a,b\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi,\{a,b\},\{a,b,c\},V\}$. Micro closed sets in V are $\phi,\{d\},\{c,d\},V$. Let $f: (U,\tau_R(X), \mu_R(X)) \to (V,\tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Then f is Micro semi continuous but not Micro g^*-continuous, since for the Micro closed set $\{d\}$ in V, $f^{-1}(\{d\}) = \{d\}$ is not Micro g^*-closed in U.

Example 3.11 Let $U = \{a,b,c,d\}$, $U/R = \{\{c\},\{d\},\{a,b\}\}$. Let $X = \{c,d\} \subseteq U$. Then, $\tau_R(X) = \{U,\phi,\{c,d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi,\{a\},\{c,d\},\{a,c,d\},U\}$. Micro g^*-closed sets in U are $\phi,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\},U$. Micro semi closed sets in U are $\phi,\{b\},\{a,b\},\{b,c,d\},U$. Let $V = \{a,b,c,d\}$, $V/R = \{\{c\},\{d\},\{a,b\}\}$. Let $Y = \{a,b\} \subseteq V$. Then, $\tau_R(Y) = \{V,\phi,\{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi,\{a\},\{a,b\},V\}$. Micro closed sets in V are $\phi,\{c,d\},\{b,c,d\},V$. Let $f: (U,\tau_R(X), \mu_R(X)) \to (V,\tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Then f is Micro g^*-continuous but not Micro semi continuous, since for the Micro closed set $\{c,d\}$ in V, $f^{-1}(\{c,d\}) = \{b,d\}$ is not Micro semi closed in U.

Remark 3.12 Micro g^*-continuous maps and Micro pre continuous maps are independent as observed from the following examples.

Example 3.13 Let $U = \{a,b,c\}$, $U/R = \{\{a\},\{b\},\{c\}\}$. Let $X = \{b,c\} \subseteq U$. Then, $\tau_R(X) = \{U,\phi\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi,\{a\},U\}$. Micro g^*-closed sets in U are

$\phi, \{b,c\}, U$. Micro pre-closed sets in U are $\phi, \{b\}, \{c\}, \{b,c\}, U$. Let $V = \{a,b,c\}$, $V/R = \{\{a\},\{b,c\}\}$. Let $Y = \{a,b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi,\}$. Let $\mu = \{a,b\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a,b\}, V\}$. Micro closed sets in V are $\phi, \{c\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be the identity map. Then f is Micro pre continuous but not Micro g^*-continuous, since for the Micro closed set $\{c\}$ in V, $f^{-1}(\{c\}) = \{c\}$ is not Micro g^*-closed in U.

Example 3.14 Let $U = \{a,b,c,d\}$, $U/R = \{\{c\},\{d\},\{a,b\}$. Let $X = \{a,b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{a,b\}, U\}$. Micro pre closed sets in U are $\phi, \{\{b\},\{c\},\{d\},\{b,c\},\{b,d\},\{c,d\},\{b,c,d\}\}, U$. Micro g^*closed sets in U are $\phi, \{c,d\}, \{a,c,d\}, \{b,c,d\}, U$. Let $V = \{a,b,c,d\}$, $V/R = \{\{c\},\{d\},\{a,b\}\}$. Let $Y = \{c\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{b\}, \{c\}, \{b,c\}, V\}$. Micro closed sets in V are $\phi, \{a,d\}, \{a,b,d\}, \{a,c,d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = c$, $f(b) = b, f(c) = a, f(d) = d$. Then f is Micro g^* -continuous but not Micro pre-continuous, since for the Micro closed set $\{a,c,d\}$ in V, $f^{-1}(\{a,c,d\}) = \{a,c,d\}$ is not Micro pre-closed in U.

Remark 3.15 Micro g^*-continuous maps and Micro α-continuous maps are independent as observed from the following examples .

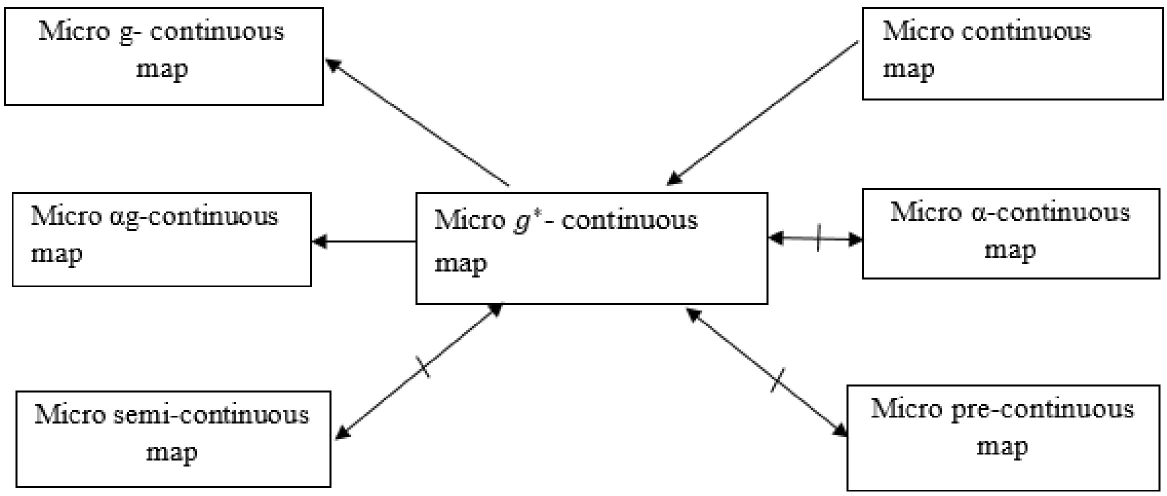
Example 3.16 Let $U = \{a,b,c,d\}$, $U/R = \{\{a\},\{c\},\{b,d\}\}$. Let $X = \{b,d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{b,d\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{b\}, \{b,d\}, U\}$. Micro α-closed sets in U are $\phi, \{a\}, \{c\}, \{d\}, \{a,c\}, \{c,d\}, \{a,c,d\}, U$. Micro g^*-closed sets in U are $\phi, \{a,c\}, \{a,b,c\}, \{a,c,d\}, U$. Let $V = \{a,b,c,d\}$, $V/R = \{\{a,b\},\{c\},\{d\}\}$. Let $Y = \{a,b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, \{a,b\}, V\}$. Micro closed sets in V are $\phi, \{c,d\}, \{b,c,d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = b$, $f(b) = a, f(c) = c, f(d) = d$. Then f is Micro α-continuous but not Micro g^*-continuous, since for the Micro closed set $\{c,d\}$ in V, $f^{-1}(\{c,d\}) = \{c,d\}$ is not Micro g^*-closed in U.

Example 3.17 Let $U = \{a,b,c\}$, $U/R = \{\{a\},\{b,c\}\}$. Let $X = \{a,b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a,b\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a,b\}, U\}$. Micro α-closed sets in U are $\phi, \{c\}, U$. Micro g^*-closed sets in U are $\phi, \{c\}, \{a,c\}, \{b,c\}, U$. Let $V = \{a,b,c\}$, $V/R = \{\{c\},\{a,b\}\}$. Let $Y = \{b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{b\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, \{b\}, \{a,b\}, V\}$. Micro closed sets in V are $\phi, \{c\}, \{a,c\}, \{b,c\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be the identity map . Then f is Micro g^*-

continuous but not Micro α-continuous, since for the Micro closed set $\{a, c\}$ in V, $f^{-1}(\{a, c\}) = \{a, c\}$ is not Micro α-closed in U.

Remark 3.18

The following diagram shows the dependency and independency relations of Micro g^*-continuous maps with already existing Micro continuous maps.



Theorem 3.19 Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y)$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is Micro g^*-continuous if and only if $f^{-1}(M)$ is Micro g^*-open in $(U, \tau_R(X), \mu_R(X))$ whenever M is Micro open in $(V, \tau_R(Y), \mu_R(Y))$.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-continuous map and M be Micro open in $(V, \tau_R(Y), \mu_R(Y))$. Then M^c is Micro closed in $(V, \tau_R(Y), \mu_R(Y))$. By hypothesis $f^{-1}(M^c)$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. i.e. $[f^{-1}(M)]^c$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Hence $f^{-1}(M)$ is Micro g^*-open in U. Conversely, suppose $f^{-1}(M)$ is Micro g^*-open in $(U, \tau_R(X), \mu_R(X))$ whenever M is Micro open in $(V, \tau_R(Y), \mu_R(Y))$. Let P be Micro closed in $(V, \tau_R(Y), \mu_R(Y))$, then P^c is Micro open in $(V, \tau_R(Y), \mu_R(Y))$. By assumption $f^{-1}(P^c)$ is Micro g^*-open in $(U, \tau_R(X), \mu_R(X))$. i.e. $[f^{-1}(P)]^c$ is Micro g^*-open in $(U, \tau_R(X), \mu_R(X))$. Then $f^{-1}(P)$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is a Micro g^*-continuous map.

Remark 3.20 The composition of two Micro g^*-continuous maps need not be a Micro g^*-continuous map as seen from the following example.

Example 3.21 Let $U = \{a, b, c\}$, $U/R = \{\{c\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{c\}, \{a, b\}, U\}$. Micro g^*-closed sets in U are $\phi, \{c\}, \{a,b\}, U$. Let $V = \{a,b,c\}$, $V/R = \{\{a\},\{b,c\}\}$. Let $Y = \{a,b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi\}$. Let $\mu = \{a,b\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a,b\}, V\}$. Micro closed sets in V are $\phi, \{c\}, V$. Micro g^*-closed sets in V are $\phi, \{c\}, \{a,c\}, \{b,c\}, V$. Let $W = \{a,b,c\}$, $W/R = \{\{c\},\{a,b\}\}$. Let $Z = \{b\} \subseteq W$. Then, $\tau_R(Z) = \{W, \phi, \{b\}\}$. Let $\mu = \{a\} \notin \tau_R(Z)$. Then, $\mu_R(Z) = \{\phi, \{a\}, \{b\}, \{a,b\}, W\}$. Micro closed sets in W are $\phi, \{c\}, \{a,c\}, \{b,c\}, W$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ and $g: (V, \tau_R(Y), \mu_R(Y)) \to (W, \tau_R(Z), \mu_R(Z))$ be the maps defined by $f(a) = b$, $f(b) = a, f(c) = c$, $g(a) = b$, $g(b) = a, g(c) = c$. Then the maps f and g are Micro g^*-continuous but their composition $g \circ f: (U, \tau_R(X), \mu_R(X)) \to (W, \tau_R(Z), \mu_R(Z))$ is not a Micro g^*-continuous map, since for the closed set $\{a,c\}$ in $(W, \tau_R(Z), \mu_R(Z))$, $(g \circ f)^{-1}(\{a,c\}) = \{a,c\}$ is not Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$.

Theorem 3.22 Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-continuous map and $g: (V, \tau_R(Y), \mu_R(Y)) \to (W, \tau_R(Z), \mu_R(Z))$ be a Micro continuous map. Then $g \circ f: (U, \tau_R(X), \mu_R(X)) \to \left(W, \tau_R(Z), \mu_R(Z)\right)$ is a Micro g^*-continuous map.

Proof: Let P be a Micro closed set in $(W, \tau_R(Z), \mu_R(Z))$. Since g is Micro continuous, $g^{-1}(P)$ is Micro closed in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro g^*-continuous, $(g \circ f)^{-1}(P) = f^{-1}\left(g^{-1}(P)\right)$ is Micro g^*-closed. Hence $g \circ f$ is a Micro g^*-continuous map.

Definition 3.23 A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro $T^*_{1/2}$- space (briefly Mic- $T^*_{1/2}$- space) if every Micro g^*-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $\left(U, \tau_R(X), \mu_R(X)\right)$.

Definition 3.24 A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a Micro ${}_\alpha T_c$-space (briefly Mic- ${}_\alpha T_c$- space) if every Micro αg-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro g^*-closed in $\left(U, \tau_R(X), \mu_R(X)\right)$.

Definition 3.25 A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a Micro ${}^*T_{1/2}$-space (briefly Mic-${}^*T_{1/2}$-space) if every Micro g-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro g^*-closed in $\left(U, \tau_R(X), \mu_R(X)\right)$.

Theorem 3.26 Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$- space then f is a Micro continuous map.

Proof: Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is a Micro g^*-continuous map, $f^{-1}(P)$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$-space, $f^{-1}(P)$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is a Micro continuous map.

Theorem 3.27 Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$- space then f is a Micro g continuous map.

Proof: Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is a Micro g^*-continuous map, $f^{-1}(P)$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $T^*_{1/2}$-space, $f^{-1}(P)$ is Micro closed. Since every Micro closed set is Micro g-closed, $f^{-1}(P)$ is Micro g-closed. Hence f is a Micro g-continuous map in $(U, \tau_R(X), \mu_R(X))$.

Theorem 3.28 Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro αg-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro $_\alpha T_c$- space then f is a Micro g^*-continuous map.

Proof: Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since every Micro g^*-closed set is Micro αg-closed and f is Micro αg-continuous, $f^{-1}(P)$ is Micro αg-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $_\alpha T_c$- space, $f^{-1}(P)$ is Micro g^*-closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is a Micro g^*-continuous map.

Theorem 3.29 Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro g-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro $^* T_{1/2}$- space, then f is Micro g^*- continuous.

Proof: Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro g-continuous, $f^{-1}(P)$ is Micro g-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro* $T_{1/2}$-space, $f^{-1}(P)$ is Micro g^*-closed. Hence f is Micro g^*-continuous.

4. Micro g^*-Irresolute Maps

In this section, the strong form Micro g^*-continuous maps, namely Micro g^*-irresolute maps is introduced and its properties are analyzed. It is shown that composition of two Micro g^*-irresolute maps is also a Micro g^*-irresolute map.

Definition 4.1 Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called a Micro g^*-irresolute map if $f^{-1}(G)$ is Micro g^*-closed in U for every Micro g^*-closed set G in V.

Example 4.2 Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{c, d\} \subseteq U$. Then, $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, U\}$. Micro g^*-closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$, $V/R = \{\{a, b\}, \{c\}, \{d\}\}$. Let $Y = \{a, b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi, \{a, b\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a\}, \{a, b\}, V\}$. Micro g^*-closed sets in V are $\phi, \{c, d\}, \{a, c, d\}, \{b, c, d\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a$, $f(b) = c, f(c) = b, f(d) = d$. Therefore for every Micro g^*-closed set P in V, $f^{-1}(P)$ is Micro g^*-closed in U. Hence f is a Micro g^*-irresolute map.

Proposition 4.3 Every Micro g^*-irresolute map is a Micro g^*-continuous map but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-continuous map. Let P be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since every Micro closed set is Micro g^*-closed and f is Micro g^*-irresolute, $f^{-1}(P)$ is Micro g^*-closed. Hence f is a Micro g^*-continuous map.

Example 4.4 Let $U = \{a, b, c\}$, $U/R = \{\{a, b\}, \{c\}\}$. Let $X = \{a, b\} \subseteq U$. Then ,$\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then, $\mu_R(X) = \{\phi, \{c\}, \{a, b\}, U\}$. Micro g^*-closed sets in U are $\phi, \{c\}, \{a, b\}, U$. Let $V = \{a, b, c\}$, $V/R = \{\{a\}, \{b, c\}\}$. Let $Y = \{a, b\} \subseteq V$. Then, $\tau_R(Y) = \{V, \phi\}$. Let $\mu = \{a, b\} \notin \tau_R(Y)$. Then, $\mu_R(Y) = \{\phi, \{a, b\}, V\}$. Micro closed sets in V are $\phi, \{c\}, V$. Micro g^*-closed sets in V are $\{\phi, \{c\}, \{a, c\}, \{b, c\}, V\}$. Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = b$, $f(b) = a, f(c) = c$. Then f is Micro g^*-continuous but not Micro g^*-irresolute, since for the Micro closed set $\{a, c\}$ in V, $f^{-1}(\{a, c\}) = \{b, c\}$ is not Micro g^*-closed in U.

Theorem 4.5 Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-irresolute map and $g: (V, \tau_R(Y), \mu_R(Y)) \rightarrow (W, \tau_R(Z), \mu_R(Z))$ be a Micro g^*-irresolute map then $g \circ f: (U, \tau_R(X), \mu_R(X)) \rightarrow (W, \tau_R(Z), \mu_R(Z))$ is a Micro g^*-irresolute map.

Proof: Let P be a Micro g^*-closed set in $(W, \tau_R(Z), \mu_R(Z))$. Since g is Micro g^*-irresolute, $g^{-1}(P)$ is Micro g^*-closed in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro g^*-irresolute, $(g \circ f)^{-1}(P) = f^{-1}(g^{-1}(P))$ is Micro g^*-closed. Hence $g \circ f$ is a Micro g^*- irresolute map.

Theorem 4.6 Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro g^*-irresolute map and $g: (V, \tau_R(Y), \mu_R(Y)) \rightarrow (W, \tau_R(Z), \mu_R(Z))$ be a Micro g^*-continuous map then $g \circ f: (U, \tau_R(X), \mu_R(X)) \rightarrow (W, \tau_R(Z), \mu_R(Z))$ is a Micro g^*- continuous map.

Proof: Let P be a Micro closed set in $(W, \tau_R(Z), \mu_R(Z))$. Since g is Micro g^*-continuous, $g^{-1}(P)$ is Micro g^*-closed in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro g^*-irresolute, $(g \circ f)^{-1}(P) = f^{-1}(g^{-1}(P))$ is Micro g^*-closed. Hence $g \circ f$ is a Micro g^*- continuous map.

Theorem 4.7 Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro g-irresolute map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro $^* T_{1/2}$- space, then f is Micro g^*-irresolute.

Proof: Let P be a Micro g^*-closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since every Micro g^*-closed set is Micro g-closed and f is Micro g-irresolute, $f^{-1}(P)$ is Micro g-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro* $T_{1/2}$-space, $f^{-1}(P)$ is Micro g^*-closed. Hence f is Micro g^*-irresolute.

Theorem 4.8 Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be a Micro αg-irresolute map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro $_\alpha T_c$- space, then f is Micro g^*-irresolute.

Proof: Let P be a Micro g^*-closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since every Micro g^*-closed set is Micro αg-closed and f is Micro αg- irresolute, $f^{-1}(P)$ is Micro αg-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro $_{\alpha}\, T_c$-space, $f^{-1}(P)$ is Micro g^*-closed. Hence f is Micro g^*-irresolute.

Conclusion:

In this article, we have introduced Micro g^*-continuous map and Micro g^*-irresolute map in Micro topological spaces. Further the fundamental properties of the defined maps are examined.

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MICRO Ψ-CONTINUOUS MAPS AND MICRO Ψ-IRRESOLUTE MAPS IN MICRO TOPOLOGICAL SPACES

T. Sowmiya, R. Anandhi and N. Balamani

Department of Mathematics,

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore-641043

Corresponding author E-mail: sowmiyathangavel1401@gmail.com,

anandhi08081985@gmail.com, nbalamani77@gmail.com

Abstract:

The aim of this chapter is to introduce Micro ψ-continuous maps and Micro ψ-irresolute maps in Micro topological spaces. Fundamental properties are derived and associations with the previously existing maps are obtained.

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Keywords: Micro continuous map, Micro ψ-continuous map, Micro ψ-irresolute map.

1. Introduction:

Rough set theory is a new mathematical approach. The notion of rough set theory was proposed by Pawlak [6].The concept of Nano topology was introduced by Carmel Richard [4]. He has defined the nano topological space with respect to a subset X of a universe U which is defined on lower, upper approximations and boundary region of X. Sakkaraiveeranan Chandrasekar [8] introduced the concepts of Micro continuous map and he also studied Micro semi-continuous and Micro pre-continuous maps in Micro topological spaces. The concept of Micro α-continuous maps was introduced by Chandrasekar and Swathi [5]. Anandhi and Balamani [1,2,3] studied the concept of Micro αg-closed sets, separation axioms and Micro αg-continuous maps and presented basic properties and theorems. Micro g-continuous map was introduced by Taha *et al.* [11]. Recently Sandhiya and Balamani [9] introduced Micro g^*-closed sets in Micro topological spaces and also Sowmiya and Balamani [10] introduced Micro ψ-closed sets in Micro topological spaces and examined their properties. In this chapter we have introduced Micro ψ-continuous maps in Micro topological spaces. Dependency and independency relations are obtained by comparing the Micro ψ-continuous maps with already existing Micro continuous maps.

2. Premilinaries:

Definition 2.1 [6] Let U be a nonempty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.That is, $L_R(X) = \bigcup_{x\in U}\{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.That is, $U_R(X)$= $\bigcup_{x\in U}\{R(x): R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$.
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$is a topology on U called the Nano topology on U with respect to X. We call $(U,\tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets and the complement of a Nano open set is called a Nano closed set.

Definition 2.3[8] Let $(U,\tau_R(X))$ be a Nano topological space. Then $\mu_R(X) = \{N \cup (N' \cap \mu): N, N' \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ and $\mu_R(X)$ satisfies the following axioms:

(i) $U, \phi \in \mu_R(X)$.

(ii) The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

(iii) The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then, $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U,\tau_R(X),\mu_R(X))$ is called Micro topological space and the elements of $\mu_R(X)$are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4[10] Let $(U,\tau_R(X),\mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro ψ-closed if $Mic\text{-}scl(A) \subseteq L$ whenever $A \subseteq L$ and L is Mic-sg-open in U.

Definition 2.5 Let $\left(U, \tau_R(X), \mu_R(X)\right)$ and $\left(V, \tau_R(Y), \mu_R(Y)\right)$ be two Micro topological spaces. A map $f: \left(U, \tau_R(X), \mu_R(X)\right) \to \left(V, \tau_R(Y), \mu_R(Y)\right)$ is called a

(i) Micro continuous map [8] if $f^{-1}(K)$ is Micro closed in U for every Micro closed set K in V.

(ii) Micro pre-continuous map [8] if $f^{-1}(K)$ is Micro pre-closed in U for every Micro closed set K in V.

(iii) Micro semi-continuous map [8] if $f^{-1}(K)$ is Micro semi-closed in U for every Micro closed set K in V.

(iv) Micro α-continuous map [5,7] if $f^{-1}(K)$ is Micro α-closed in U for every Micro closed set K in V.

(v) Micro αg-continuous map [3] if $f^{-1}(K)$ is Micro αg-closed in U for every Micro closed set K in V.

(vi) Micro g-continuous map [11] if $f^{-1}(K)$ is Micro g-closed in U for every Micro closed set K in V .

3. Micro ψ-continuous maps and its properties:

In this section we introduce Micro ψ-continuous maps in Micro topological spaces and derive the dependency and independency relations of newly defined map with already existing Micro continuous maps. Also we derive the composition of mappings with respect to the newly defined Micro ψ-continuous maps.

Definition 3.1 Let $\left(U, \tau_R(X), \mu_R(X)\right)$ and $\left(V, \tau_R(Y), \mu_R(Y)\right)$ be two Micro topological spaces. A map $f: \left(U, \tau_R(X), \mu_R(X)\right) \to \left(V, \tau_R(Y), \mu_R(Y)\right)$ is called Micro ψ-continuous if $f^{-1}(K)$ is Micro ψ-closed in U for every Micro closed set K in V.

Example 3.2 Let $U = \{a, b, c, d\}$,$U/R = \{\{a\}, \{c\}, \{b, d\}\}$.Let$X$=$\{b, d\} \subseteq U$. Then $\tau_R(X)$=$\{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$.Micro ψ-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$.Let $V = \{a, b, c, d\}$,$V/R = \{\{a\}, \{b\}, \{c, d\}\}$.Let Y=$\{c, d\} \subseteq V$.Then$\tau_R(Y)$=$\{V, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$.Then $\mu_R(Y) = \{V, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro closed sets in V are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, V$. Let $f: \left(U, \tau_R(X), \mu_R(X)\right) \to \left(V, \tau_R(Y), \mu_R(Y)\right)$ be a map defined by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$. Therefore for every Micro closed set K in V, $f^{-1}(K)$ is Micro ψ-closed in U. Hence f is a Micro ψ-continuous map.

Proposition 3.3 Every Micro continuous map is Micro ψ-continuous but not conversely.

Proof: Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ be a Micro continuous map. Let K be a Micro closed set in $(V,\tau_R(Y),\mu_R(Y))$. Since f is Micro continuous, $f^{-1}(K)$ is Micro closed in U. Since every Micro closed set is Micro ψ-closed, $f^{-1}(K)$ is Micro ψ-closed. Hence f is Micro ψ-continuous.

Example 3.4 Let $U = \{a,b,c,d\}, U/R = \{\{a\},\{b\},\{c,d\}\}$.Let$X$=$\{c,d\} \subseteq U$.Then $\tau_R(X)$=$\{U,\phi,\{c,d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U,\phi,\{a\},\{c,d\},\{a,c,d\}\}$. Micro closed sets in U are $\phi,\{b\},\{a,b\},\{b,c,d\},U$. Micro ψ-closed sets in U are $\phi,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\},U$.Let $V = \{a,b,c,d\}$,$V/R = \{\{c\},\{d\},\{a,b\}\}$.Let Y=$\{a,b\} \subseteq V$. Then $\tau_R(Y)$=$\{V,\phi,\{a,b\}\}$. Let $\mu = \{d\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V,\phi,\{d\},\{a,b\},\{a,b,d\}\}$. Micro closed sets in V are $\phi,\{c\},\{c,d\},\{a,b,c\},V$. Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ be a map defined by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$.Then f is Micro ψ-continuous but not Micro continuous, since for the Micro closed set $\{c,d\}$ in V, $f^{-1}(\{c,d\}) = \{b,d\}$ is not Micro closed in U.

Proposition 3.5 Every Micro semi-continuous map is Micro ψ-continuous but not conversely.

Proof: Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ be a Micro semi-continuous map. Let K be a Micro closed set in $(V,\tau_R(Y),\mu_R(Y))$. Since f is Micro semi continuous, $f^{-1}(K)$ is Micro semi closed in U. Since every Micro semi-closed set is Micro ψ-closed, $f^{-1}(K)$ is Micro ψ-closed. Therefore f is Micro ψ-continuous.

Example 3.6 Let $U = \{a,b,c,d\}$,$U/R = \{\{a\},\{c\},\{b,d\}\}$.Let$X$=$\{b,d\} \subseteq U$.Then $\tau_R(X)$=$\{U,\phi,\{b,d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$.Micro semi-closed sets in U are $\phi,\{a\},\{c\},\{d,\{a,c\},\{a,d\},\{a,c,d\},U$. Micro ψ-closed sets in U are $\phi,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\},U$.Let $V = \{a,b,c,d\}$,$V/R = \{\{c,d\},\{a,b\}\}$.Let Y=$\{a,b,c\} \subseteq V$. Then $\tau_R(Y)$=$\{V,\phi,\{a,b\}\}$. Let $\mu = \{c\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V,\phi,\{c\},\{a,b\},\{a,b,c\}\}$. Micro closed sets in V are $\phi,\{d\},\{c,d\},\{a,b,d\},V$. Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ be a map defined by $f(a) = a, f(b) = b, f(c) = d, f(d) = c$.Then f is Micro ψ-continuous but not Micro semi continuous, since for the Micro closed set $\{a,b,d\}$ in V, $f^{-1}(\{a,b,d\}) = \{a,b,c\}$ is not Micro semi-closed in U.

Proposition 3.7 Every Micro α-continuous map is Micro ψ-continuous but not conversely.

Proof: Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ be a Micro α-continuous map. Let K be a Micro closed set in $(V,\tau_R(Y),\mu_R(Y))$. Since f is Micro α-continuous, $f^{-1}(K)$ is Micro α-closed in U.Since every Micro α- closed set is Micro ψ-closed, $f^{-1}(K)$ is Micro ψ-closed. Therefore f is Micro ψ-continuous**.**

Example 3.8 Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{c\}, \{b, d\}$. Let$X = \{b, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$.Micro α-closed sets in$U$ are $\phi, \{c\}, \{a, c\}, \{b, c, d\}, U$.Micro ψ-closed sets in U are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, U$.Let$V = \{a, b, c, d\}, V/R = \{\{a\}, \{b\}, \{c, d\}\}$.Let Y=$\{c, d\} \subseteq V$.Then $\tau_R(Y)$=$\{V, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro closed sets in V are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, V$.Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = d, f(b) = c, f(c) = b, f(d) = a$.Then f is Micro ψ-continuous but not Micro α-continuous, since for the Micro closed set $\{a, b\}$ in V, $f^{-1}(\{a, b\}) = \{c, d\}$ is not Micro α-closed in U.

Remark 3.9 Micro ψ-continuous maps and Micro pre-continuous maps are independent as observed from the following examples.

Example 3.10 Let $U = \{a, b, c, d\}$,$U/R = \{\{a\}, \{b\}, \{c, d\}\}$.Let $X = \{c, d\} \subseteq U$.Then $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro pre closed sets in U are $\phi, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, U$. Micro ψ-closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$,$V/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $Y = \{c\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{b\}, \{c\}, \{b, c\}\}$. Micro closed sets in V are $\phi, \{a, d\}, \{a, b, d\}, \{a, c, d\}, V$.Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a, f(b) = d, f(c) = b, f(d) = c$.Then f is Micro ψ-continuous but not Micro pre-continuous, since for the Micro closed set $\{a, c, d\}$ in V, $f^{-1}(\{a, c, d\}) = \{a, b, d\}$ is not Micro pre-closed in U.

Example 3.11 Let $U = \{a, b, c, d\}$,$U/R = \{\{c\}, \{d\}, \{a, b\}\}$.Let $X = \{a, b\} \subseteq U$.Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let$\mu = \{d\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Micro pre closed sets in U are $\phi, \{b\}, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, U$. Micro ψ-closed sets in $U$are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$, $V/R = \{\{a\}, \{c\}, \{b, d\}$. Let $Y = \{b, d\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro closed sets in V are $\phi, \{c\}, \{a, c\}, \{b, c, d\}, V$.Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be the identity map. Then f is Micro pre-continuous but not Micro ψ-continuous, since for the Micro closed set $\{a, c\}$ in V, $f^{-1}(\{a, c\}) = \{a, c\}$ is not Micro ψ-closed in U.

Remark 3.12 Micro ψ-continuous maps and Micro g-continuous maps are independent as observed from the following examples.

Example 3.13 Let $U = \{a, b, c, d\}, U/R = \{\{a, b\}, \{c, d\}$.Let $X = \{a, b, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}$. Micro g-closed sets in U are $\phi,\{d\},\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\},U$. Micro ψ-closed sets in $U$are $\phi,\{d\},\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},U$. Let $V=\{a,b,c,d\}$,$V/R=\{\{a\},\{c\},\{b,d\}\}$. Let $Y=\{b,d\}\subseteq V$. Then $\tau_R(Y)=\{V,\phi,\{b,d\}\}$. Let $\mu=\{a\}\notin\tau_R(Y)$. Then $\mu_R(Y)=\{V,\phi,\{a\},\{b,d\},\{a,b,d\}\}$. Micro closed sets in V are $\phi,\{c\},\{a,c\},\{b,c,d\},V$. Let $f:(U,\tau_R(X),\mu_R(X))\to(V,\tau_R(Y),\mu_R(Y))$ be a map defined by $f(a)=a,f(b)=b,f(c)=d,f(d)=c$.Then f is Micro g-continuous but not Micro ψ-continuous, since for the Micro closed set $\{b,c,d\}$ in V, $f^{-1}(\{b,c,d\})=\{b,c,d\}$ is not Micro ψ-closed in U.

Example 3.14 Let $U=\{a,b,c,d\},U/R=\{\{a\},\{c\},\{b,d\}$.Let $X=\{b,d\}\subseteq U$. Then $\tau_R(X)=\{U,\phi,\{b,d\}\}$. Let$\mu=\{a\}\notin\tau_R(X)$. Then $\mu_R(X)=\{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$.Micro g-closed sets in U are $\phi,\{a\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\},U$. Micro ψ-closed sets in U are $\phi,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\},U$.Let $V=\{a,b,c,d\}$,$V/R=\{\{a,b\},\{c,d\}$.Let $Y=\{a,b,c\}\subseteq V$.Then$\tau_R(Y)=\{V,\phi,\{a,b\}\}$. Let $\mu=\{c\}\notin\tau_R(Y)$. Then $\mu_R(Y)=\{V,\phi,\{c\},\{a,b\},\{a,b,c\}$. Micro closed sets in V are $\phi,\{d\},\{c,d\},\{a,b,d\},V$. Let $f:(U,\tau_R(X),\mu_R(X))\to(V,\tau_R(Y),\mu_R(Y))$ be a map defined by $f(a)=b,f(b)=a,f(c)=d,f(d)=c$. Then f is Micro ψ-continuous but not Micro g-continuous, since for the Micro closed set $\{d\}$ in V, $f^{-1}(\{d\})=\{c\}$ is not Micro g-closed in U.

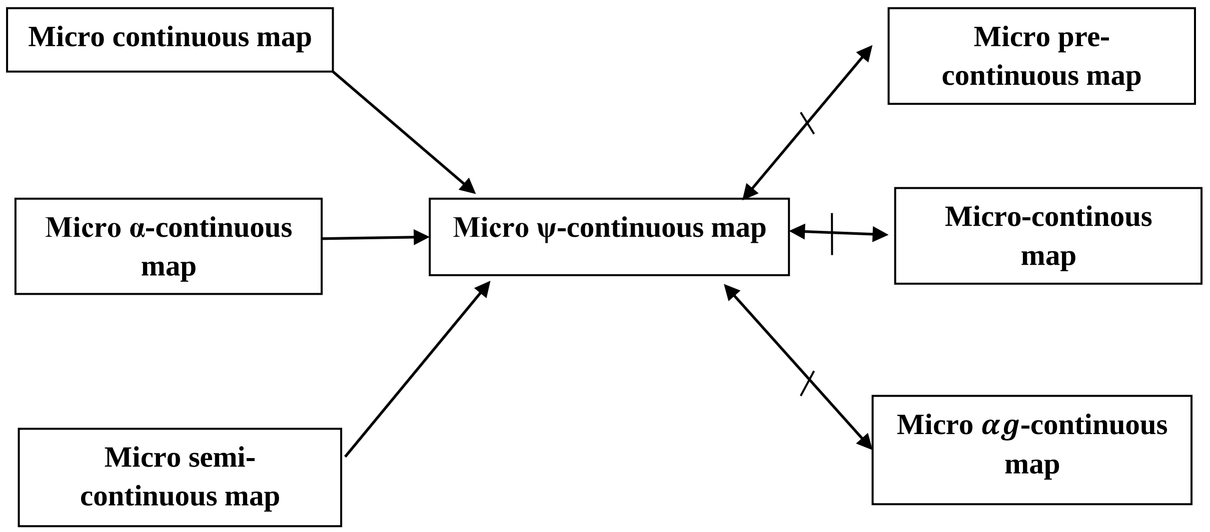
Remark 3.15 Micro ψ-continuous maps and Micro αg-continuous maps are independent as observed from the following examples.

Example 3.16 Let $U=\{a,b,c,d\}$,$U/R=\{\{c\},\{d\},\{a,b\}\}$.Let $X=\{a,b\}\subseteq U$. Then$\tau_R(X)=\{U,\phi,\{a,b\}\}$. Let $\mu=\{d\}\notin\tau_R(X)$. Then $\mu_R(X)=\{U,\phi,\{d\},\{a,b\},\{a,b\},\{a,b,d\}\}$.Micro αg-closed sets in U are $\phi,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\},U$. Micro ψ-closed sets in U are $\phi,\{c\},\{d\},\{a,b\},\{c,d\},\{a,b,c\},U$. Let $V=\{a,b,c,d\}$,$V/R=\{\{a\},\{b\},\{c,d\}\}$.Let $Y=\{c,d\}\subseteq V$.Then$\tau_R(Y)=\{V,\phi,\{c,d\}\}$.Let $\mu=\{a\}\notin\tau_R(Y)$. Then $\mu_R(Y)=\{V,\phi,\{a\},\{c,d\},\{a,c,d\}\}$. Micro closed sets in V are $\phi,\{b\},\{a,b\},\{b,c,d\},V$.Let $f:(U,\tau_R(X),\mu_R(X))\to(V,\tau_R(Y),\mu_R(Y))$ be a map defined by $f(a)=a,f(b)=c,f(c)=b,f(d)=d$. Then f is Micro αg-continuous but not Micro ψ-continuous, since for the Micro closed set $\{a,b\}$ in V, $f^{-1}(\{a,b\})=\{a,c\}$ is not Micro ψ-closed in U.

Example 3.17 Let $U=\{a,b,c\}$,$U/R=\{\{a\},\{b,c\}\}$.Let $X=\{a,b\}\subseteq U$,$\tau_R(X)=\{U,\phi,\{a\}\}$. Let $\mu=\{b\}\notin\tau_R(X)$. Then $\mu_R(X)=\{U,\phi,\{a\},\{b\},\{a,b\}\}$. Micro αg-closed sets in U are $\phi,\{c\},\{a,c\},\{b,c\},U$. Micro ψ-closed sets in U are $\phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\},U$. Let $V=\{a,b,c\}$,$V/R=\{\{c\},\{a,b\}\}$. Let $Y=\{a,b\}\subseteq V$. Then $\tau_R(Y)=\{V,\phi,\{a,b\}\}$. Let $\mu=\{c\}\notin\tau_R(Y)$. Then $\mu_R(Y)=\{V,\phi,\{c\},\{a,b\}\}$. Micro closed sets in V are $\phi,\{c\},\{a,b\},V$. Let $f:(U,\tau_R(X),\mu_R(X))\to(V,\tau_R(Y),\mu_R(Y))$ be a map defined by $f(a)=b,f(b)=c,f(c)=a$.

Then f is Micro ψ-continuous but not Micro αg-continuous, since for the Micro closed set $\{c\}$ in V, $f^{-1}(\{c\}) = \{b\}$ is not Micro αg-closed in U.

Remark 3.18 The following diagram shows the dependency and independency relations of Micro ψ-continuous map with already existing various Micro continuous maps.



Remark 3.19 The composition of two Micro ψ-continuous maps need not be a Micro ψ-continuous map as seen from the following example.

Example 3.20 Let $U = \{a,b,c,d\}, U/R = \{\{a\},\{b\},\{c,d\}\}$. Let $X = \{c,d\} \subseteq U$.Then $\tau_R(X) = \{U,\phi,\{c,d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U,\phi,\{a\},\{c,d\}, \{a,c,d\}\}$.Micro ψ-closed sets in U are $\phi,\{b\},\{a,b\}, \{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\},U$. Let $V = \{a,b,c,d\}$,$V/R = \{\{c\},\{d\},\{a,b\}\}$. Let $Y = \{a,b\} \subseteq V$. Then $\tau_R(Y) = \{V,\phi,\{a,b\}\}$. Let $\mu = \{d\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V,\phi,\{d\},\{a,b\},\{a,b\},\{a,b,d\}\}$.Micro closed sets in V are $\phi,\{c\},\{c,d\},\{a,b,c\},V$. Micro ψ-closed sets in V are $\phi,\{c\},\{d\},\{a,b\},\{c,d\},\{a,b,c\},V$. Let $W = \{a,b,c,d\}$,$W/R = \{\{a\},\{c\},\{b,d\}\}$. Let $Z = \{b,d\} \subseteq W$,$\tau_R(Z) = \{W,\phi,\{b,d\}\}$. Let $\mu = \{b\} \notin \tau_R(Z)$.Then $\mu_R(Z) = \{W,\phi,\{b\},\{b,d\}\}$. Micro closed sets in W are $\phi,\{a,c\}, \{a,c,d\},W$. Let $f: (U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ and $g: (V,\tau_R(Y),\mu_R(Y)) \to (W,\tau_R(Z),\mu_R(Z))$ be the maps defined by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$ and $g(a) = a, g(b) = c, g(c) = d, g(d) = b$. Then both $f$and g are Micro ψ-continuous but their composition $g \circ f: (U,\tau_R(X),\mu_R(X)) \to (W,\tau_R(Z),\mu_R(Z))$ is not Micro ψ-continuous, since for the Micro closed set $\{c\}$ in W, $(g \circ f)^{-1}(\{a,c\}) = f^{-1}(g^{-1}(\{a,c\})) = f^{-1}\{a,b\} = \{a,c\}$ is not Micro ψ-closed in U.

Theorem 3.21 Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-continuous map and $g: (V, \tau_R(Y), \mu_R(Y)) \to (W, \tau_R(Z), \mu_R(Z))$ be a Micro continuous map, then $g \circ f: (U, \tau_R(X), \mu_R(X)) \to (W, \tau_R(Z), \mu_R(Z))$ is a Micro ψ-continuous map.

Proof: Let K be a Micro closed set in $(W, \tau_R(Z), \mu_R(Z))$. Since g is Micro continuous, $g^{-1}(K)$ is Micro closed in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro ψ-continuous, $(g \circ f)^{-1}(K) = f^{-1}(g^{-1}(K))$ is Micro ψ-closed. Hence $g \circ f$ is Micro ψ-continuous.

Definition 3.22 A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a Micro semi-$T_{1/3}$ -space (briefly Mic-semi-$T_{1/3}$ -space) if every Micro ψ-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$.

Definition 3.23 A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a Micro semi-$T_{1/2}$ -space (briefly Mic-semi-$T_{1/2}$ -space) if every Micro sg-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$.

Definition 3.24 A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a Micro ${}_{\psi}T_c$ -space (briefly Mic- ${}_{\psi}T_c$ -space) if every Micro ψ-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$.

Theorem 3.25 Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$ -space then f is a Micro semi continuous.

Proof: Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro ψ-continuous, $f^{-1}(K)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$ space, $f^{-1}(K)$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is Micro semi-continuous.

Theorem 3.26 Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/2}$ -space then f is Micro semi continuous.

Proof: Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro ψ- continuous, $f^{-1}(K)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Since every Micro ψ-closed set is Micro sg-closed and $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/2}$-space, $f^{-1}(K)$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is Micro semi-continuous.

Theorem 3.27 Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-continuous map and if $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_{\psi}T_c$-space then f is Micro continuous.

Proof: Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Since f is Micro ψ- continuous, $f^{-1}(K)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Since $(U, \tau_R(X), \mu_R(X))$ is a Micro ${}_{\psi}T_c$-space, $f^{-1}(K)$ is Micro closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is Micro continuous.

Theorem 3.28 Let $(U, \tau_R(X), \mu_R(X))$and $(V, \tau_R(Y), \mu_R(Y))$ be any two maps. Then $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is Micro ψ-continuous if and only if $f^{-1}(B)$is Micro ψ-open in $(U, \tau_R(X), \mu_R(X))$ whenever B is a Micro open set in $(V, \tau_R(Y), \mu_R(Y))$.

Proof: Let f be a Micro ψ-continuous map and B be Micro open in $(V, \tau_R(Y), \mu_R(Y))$. Then B^c is Micro closed in$(V, \tau_R(Y), \mu_R(Y))$. By hypothesis $f^{-1}(B^c)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$, i.e.,$[f^{-1}(B)]^c$ is a Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$. Hence $f^{-1}(B)$ is a Micro ψ-open in $(U, \tau_R(X), \mu_R(X))$. Conversely, suppose $f^{-1}(B)$ is a Micro ψ-open set in $(U, \tau_R(X), \mu_R(X))$ whenever B is Micro open in $(V, \tau_R(Y), \mu_R(Y))$. Let H be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$. Then H^c is Micro open set in $(V, \tau_R(Y), \mu_R(Y))$. By assumption $f^{-1}(H^c)$is Micro ψ-open in $(U, \tau_R(X), \mu_R(X))$, i.e., $[f^{-1}(H)]^c$ is a Micro ψ-open set in $(U, \tau_R(X), \mu_R(X))$. Then $f^{-1}(H)$ is Micro ψ-closed in $(U, \tau_R(X), \mu_R(X))$. Hence f is a Micro ψ-continuous map.

4. Micro ψ-Irresolute Maps and its Properties:

This section presents the definition and properties of Micro ψ-irresolute maps.

Definition 4.1 A map $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ is called Micro ψ-irresolute if $f^{-1}(K)$ is Micro ψ-closed in U for every Micro ψ- closed set K in V.

Proposition 4.2 Every Micro ψ-irresolute map is Micro ψ-continuous but not conversely.

Proof: Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a Micro ψ-irresolute map. Let K be a Micro closed set in $(V, \tau_R(Y), \mu_R(Y))$.Since every Micro closed set is Micro-ψ-closed and f is Micro ψ-irresolute,$f^{-1}(K)$ is Micro ψ-closed. Hence f is Micro ψ-continuous.

Example 4.3 Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b\}, \{c, d\}$.Let $X = \{c, d\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$. Micro closed sets in U are $\phi, \{b\}, \{a, b\}, \{b, c, d\}, U$. Micro ψ-closed sets in $U$are $\phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}\{a, b, d\}, \{b, c, d\}, U$. Let $V = \{a, b, c, d\}$,$V/R = \{\{c\}, \{d\}, \{a, b\}\}$.Let $Y = \{a, b\} \subseteq V$. Then $\tau_R(Y) = \{V, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(Y)$. Then $\mu_R(Y) = \{V, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Micro closed sets in V are $\phi, \{c\}, \{c, d\}, \{a, b, c\}, V$. Micro ψ-closed sets in V are $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, V$. Let $f: (U, \tau_R(X), \mu_R(X)) \to (V, \tau_R(Y), \mu_R(Y))$ be a map defined by $f(a) = a, f(b) = c, f(c) = b, f(d) = d$. Then f is Micro ψ-continuous but not Micro ψ-irresolute, since for the Micro ψ-closed set $\{d\}$ in V, $f^{-1}(\{d\}) = \{d\}$ is not Micro ψ-closed in U.

Theorem 4.4 Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ be a Micro ψ-irresolute map and $g:(V,\tau_R(Y),\mu_R(Y)) \to (W,\tau_R(Z),\mu_R(Z))$ be a Micro ψ-irresolute map then $g \circ f:(U,\tau_R(X),\mu_R(X)) \to (W,\tau_R(Z),\mu_R(Z))$ is a Micro ψ-irresolute map.

Proof: Let K be a Micro ψ-closed set in $(W,\tau_R(Z),\mu_R(Z))$. Since g is Micro ψ-irresolute, $g^{-1}(K)$ is Micro ψ-closed in $(V,\tau_R(Y),\mu_R(Y))$. Since f is Micro ψ-irresolute, $(g \circ f)^{-1}(K) = f^{-1}(g^{-1}(K))$ is Micro ψ-closed. Hence $g \circ f$ is Micro ψ-irresolute.

Theorem 4.5 Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ be a Micro ψ-irresolute map and $g:(V,\tau_R(Y),\mu_R(Y)) \to (W,\tau_R(Z),\mu_R(Z))$ be a Micro ψ-continuous map then $g \circ f:(U,\tau_R(X),\mu_R(X)) \to (W,\tau_R(Z),\mu_R(Z))$ is a Micro ψ-continuous map.

Proof: Let K be a Micro closed set in $(W,\tau_R(Z),\mu_R(Z))$. Since g is Micro ψ-continuous, $g^{-1}(K)$ is Micro ψ-closed in $(V,\tau_R(Y),\mu_R(Y))$. Since f is Micro ψ-irresolute, $(g \circ f)^{-1}(K) = f^{-1}(g^{-1}(K))$ is Micro ψ-closed. Hence $g \circ f$ is Micro ψ-continuous.

Theorem 4.6 Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ be a Micro ψ-irresolute map and if $(U,\tau_R(X),\mu_R(X))$ is Micro semi-$T_{1/3}$ -space then f is a Micro semi continuous map.

Proof: Let K be a Micro closed set in $(V,\tau_R(Y),\mu_R(Y))$. Since every Micro closed set is Micro ψ-closed and f is Micro ψ-irresolute, $f^{-1}(K)$ is Micro ψ-closed in $(U,\tau_R(X),\mu_R(X))$. Since $(U,\tau_R(X),\mu_R(X))$ is a Micro semi-$T_{1/3}$-space, $f^{-1}(K)$ is Micro semi-closed in $(U,\tau_R(X),\mu_R(X))$. Hence f is Micro semi-continuous.

Theorem 4.7 Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ be a Micro ψ-irresolute map and if $(U,\tau_R(X),\mu_R(X))$ is Micro semi-$T_{1/2}$ -space then f is a Micro semi continuous map.

Proof: Let K be a Micro closed set in $(V,\tau_R(Y),\mu_R(Y))$. Since every Micro closed set is Micro ψ-closed and f is Micro ψ-irresolute, $f^{-1}(K)$ is Micro ψ-closed in $(U,\tau_R(X),\mu_R(X))$. Since every Micro ψ-closed set is Micro sg-closed and $(U,\tau_R(X),\mu_R(X))$ is a Micro semi-$T_{1/2}$ -space, $f^{-1}(K)$ is Micro semi-closed in $(U,\tau_R(X),\mu_R(X))$. Hence f is Micro semi-continuous.

Theorem 4.8 Let $f:(U,\tau_R(X),\mu_R(X)) \to (V,\tau_R(Y),\mu_R(Y))$ be a Micro ψ-irresolute map and if $(U,\tau_R(X),\mu_R(X))$ is a Micro ${}_{\psi}T_c$ -space then f is a Micro continuous map.

Proof: Let K be a Micro closed set in $(V,\tau_R(Y),\mu_R(Y))$. Since every Micro closed set is Micro ψ-closed and f is Micro ψ-irresolute, $f^{-1}(K)$ is Micro ψ-closed in $(U,\tau_R(X),\mu_R(X))$. Since $(U,\tau_R(X),\mu_R(X))$ is a Micro ${}_{\psi}T_c$-space, $f^{-1}(K)$is Micro closed in $(U,\tau_R(X),\mu_R(X))$. Hence f is Micro continuous.

Conclusion:

The study of Micro ψ-continuous maps and Micro ψ-irresolute maps in Micro topological spaces have been initiated in this article. We have presented the definition of Micro ψ-continuous maps and Micro ψ-irresolute maps. Later, we have derived the vital properties and interrelations are obtained substantially with counter examples.

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STATISTICAL ANALYSIS OF P-100 LATENCY VARIABILITY IN CLINICAL VISUAL EVOKED POTENTIAL STUDY

N. Sivanandan

Department of Electronics,

PSG College of Arts and Science, Coimbatore, Tamil Nadu, India

Corresponding author E-mail: vsivanandannanjan@gmail.com

Introduction:

Visual evoked potential (VEP) is a important tool to measure the conduction between eye retina and occipital lobes in the diagnosis of human optic nerves in neurological disorder. Like any other neurological test, exquisite attention to the technical details, acquisition of reproducible and reliable waveform, proper interpretation based on laboratory control values and correlationwith the clinical picture are essential for optimal utilization of this technique. It is very much interest to measure the reliable positive-100 (P-100) latency measurement for interpretation of VEP.

This work focused on investigating 20 normal subjects in a clinical neurology laboratory. There are four different types of stimulations are recorded for each subject. Slight changes seen statistically when compared to the real time clinical VEP record. A general error chances to produce while preparing a test in a specific measurement in P-100 latency. The statistical analysis has shown interesting, benefit outputs for clinical standardization in the routine study procedure. Based on the integrity of studio with high volume of averaging test, few artifacts of some troubles may be avoided by using VEP protocol techniques to ask a few specific measurement techniques.

Using pattern reversal or flash stimulation, there will be some chances to increase metabolism in the subject visual area. The VEP procedures in subjects with well version cortical lesions, gives extra information regarding generator sources. For less than 5 years children's, the black and white the pattern reversal stimulus is produced and if the waveforms are not stimulatable, then a strobe flash light VEP should be followed in the next stage.

Experimental methods:

Experiments were carried with 20 adult patients and all the test procedures were monitored with neuro diagnostic study preparation test laboratory in the Neurology department (darkened, sound attenuated room).at Royal care Hospital in the department of clinical Neurophysiology.. This work focused on investigating 20 normal subjects in a clinical neurology laboratory. Table- 1 showed the N-75 latency values for 20 normal adult subject's data. Table -2 showed P-100 latency values for 20 normal adult subject's data.

Table 1: Multi protocol VEP stimulation N-75 Latency values (ms)

Sr. No.	**Pattern Reversal**	**Flash Stimulation**	**Pattern Reversal using LED Goggles**	**Flash using LED Goggles**
1	73.0	81.2	79.9	86.8
2	75.8	89.5	81.4	85.4
3	76.0	79.2	85.7	85.0
4	79.6	80.2	78.9	81.0
5	71.1	74.6	78.4	81.5
6	76.9	77.3	71.9	80.6
7	69.9	71.1	80.3	80.1
8	72.7	76.2	80.9	87.5
9	81.0	87.9	89.3	86.9
10	71.5	76.9	80.4	86.1
11	69.9	70.8	72.9	80.5
12	77.1	79.9	80.1	87.4
13	78.0	75.7	76.5	77.5
14	75.9	78.6	80.3	81.4
15	89.0	87.5	77.6	78.5
16	78.9	77.4	80.3	79.5
17	78.5	87.4	79.6	80.6
18	77.8	76.4	79.0	79.9
19	78.1	79.1	79.5	80.6
20	71.0	76.2	80.0	81.2

Table 2: Multi protocol VEP stimulation P-100 Latency values (ms)

Sr. No.	Pattern Reversal	Flash Stimulation	Pattern Reversal using LED Goggles	Flash using LED Goggles
1	99.0	102.6	109.8	109.9
2	106.0	107.8	99.9	109.5
3	96.0	99.2	95.7	95.0
4	99.6	90.2	98.9	91.0
5	91.0	94.6	98.8	91.5
6	96.6	97.3	91.9	100.2
7	99.0	91.9	100.3	100.9
8	92.7	106.2	100.9	107.5
9	91.0	107.9	109.3	106.9
10	91.5	96.9	90.8	96.1
11	99.9	100.8	102.9	100.5
12	107.1	99.9	100.1	107.4
13	108.0	105.7	106.5	97.5
14	105.9	108.6	100.3	101.4
15	99.0	97.5	107.6	108.5
16	98.9	97.4	100.3	109.5
17	108.5	107.4	109.6	100.6
18	107.9	96.4	99.0	109.9
19	108.1	109.1	109.5	100.6
20	101.0	106.2	90.0	101.2

Table 3: Multi protocol VEP stimulation N-145 Latency values (ms)

Sr. No.	Pattern Reversal	Flash Stimulation	Pattern Reversal using LED Goggles	Flash using LED Goggles
1	150.6	150.9	149.2	156.7
2	146.0	147.8	149.9	139.5
3	136.0	149.2	145.7	145.8
4	139.6	140.2	138.9	141.0
5	141.0	144.6	138.8	141.5
6	146.6	147.3	141.9	150.2
7	149.0	131.9	140.3	145.8
8	132.7	146.2	140.9	147.5
9	141.0	147.9	139.3	136.9
10	131.5	146.9	140.8	136.1
11	139.9	140.8	142.9	140.5
12	147.1	149.9	140.1	147.4
13	138.0	145.7	146.5	137.5
14	145.9	148.6	140.3	141.4
15	139.0	137.5	147.6	148.5
16	138.9	147.4	140.3	139.5
17	148.5	147.4	149.6	140.6
18	147.9	146.4	139.0	149.9
19	128.1	129.1	139.5	150.6
20	151.0	146.2	140.0	151.2

In DWT, let us consider the discrete functions of four points in VEP study, i.e., f(0),f(1),f(2) and f(3).Where f(0) is the checker board pattern reversal, f(1) is the checker board flash, f(2) is the LED Goggles pattern reversal and f(3) is LED Goggles flash stimulation. These four points to be considered as f(0)=1,f(1)=4,f(2)=-3 and f(3)=0.Because m=4,j=2 and with jo=0 and summations are performed over x=0,1,2,3,j=0,1 and k=0for j=0 or k=0,1 for j=1.

We will use the Hear scaling and wave let functions and assumes that the four samples of f(x) are distributed over the support of the basic function, which is in width, substituting the four samples shown in Table (1), Table (2) and Table (3), the following 20 sample subject's data are processed and producing reliable latency measurement with different types of retinal waves as shown Table (4).

Table 4: Reliable N-75, P-100 and N-145 Latency values

Sr. No.	N-75 (ms)	P-100 (ms)	N-145 (ms)
1	80.2	105.3	151.8
2	80.0	105.8	145.8
3	82.2	96.4	144.1
4	79.9	94.9	139.9
5	76.4	93.9	141.4
6	76.6	96.5	146.5
7	75.3	98.0	141.7
8	87.8	101.8	141.8
9	86.2	103.7	138.1
10	78.7	93.8	138.4
11	73.5	101.0	141.7
12	81.1	103.6	143.7
13	76.9	104.4	142
14	79.0	104.0	140.8
15	83.1	103.1	148.0
16	79.0	101.5	139.9
17	81.5	106.5	145.1
18	78.2	103.3	144.4
19	79.3	106.8	145.0
20	77.1	99.6	145.6

From the experimental results, there is a mild variation in the P-100 latency measurement in the statistical analysis studies when compared to real time studies. For example the real time studies (Table 5) in S. No 5and 6 subject's data are comparedto statistical subject's data (Table 4) showed 98 % effective measurement in P-100 latency measurement.

Table 5: Real time subject's data in clinical neurophysiology laboratory

Sr. No.	N-75 (ms)	P-100 (ms)	N-145 (ms)
1	79.8	107.7	152.8
2	84.0	105.0	144.9
3	89.2	97.5	145.0
4	79.9	94.9	139.9
5	76.5	93.8	141.5
6	76.4	96.4	146.7
7	85.3	98.9	131.8
8	72.3	111.8	146.8
9	87.0	102.9	128.1
10	78.7	92.8	138.4
11	73.9	101.9	139.7
12	81.2	109.6	153.7
13	86.9	114.4	152
14	89.0	114.7	130.8
15	93.1	103.9	128.0
16	80.0	101.6	140.6
17	85.5	116.5	155.1
18	78.8	106.3	144.9
19	78.7	116.8	150.0
20	79.7	111.0	155.6

Conclusion:

In many circumstances, any one of the single protocol will be appropriate in the clinical laboratory. By limiting this routine study naturally associated in clinical study procedure along with additional protocols should be incorporated to meet the clinical standardizations. Also to ensure the similarity of study results should ensure similar potentials across laboratories and each laboratory must have proper standard normative values using its own protocol recording and technical recommendations. However, there is a slight variation in the P-100 latency value in the statistical analysis studies when compared to real time studies because of patient cooperation's and environmental laboratory circumstances. For our preliminary statistical analysis of high quality multi protocol VEP recordings of P-100 latency measurement methods, this techniqueis 98% effective in reliable detecting measurement.

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RECENT ADVANCES IN MATHEMATICS FOR ALLIED SCIENCES

Jyoti Bhate, Manoj Tapare*, Sujit Mane,
Prakash Chikte, Deepak Shelke and Rahul Kamble
Amruteshwar Arts, Commerce and Science College, Vinzar, Pune - 412213
*Corresponding author E-mail: mrtapare1234@gmail.com

Introduction:

Mathematics is fundamental subject and it is backbone of all other branches of the science. Mathematics is an important tool of formulating any real time problem into theoretical model. Many studies are available [1] on impact and necessity mathematics education.

Mathematical model of any problem helps to study, analyze and solve the problem. It sets the standard procedure to handle similar problems in future. Mathematical modeling of the problem helps to compare the problem with previous ones. One can easily study the effect of change in one or more parameters on overall output of the model. Mathematical modeling also facilitates the use of computers and programming to solve the problem. This reduces the error and leads us to the more accurate solution of the problem.

In recent years, mathematical models [2] are seen to be very effective in handling many real time issues like network security, data science, weather forecasting, epidemic analysis etc. On each of these topics, one can get different scientific articles, which explain the models in detail. Apart from it, the students and the researchers studying the pure mathematics are hardly aware of its applications. On the other hand researchers those who are using mathematical models to solve their problem, faces many difficulties due to their less mathematical background. Many high school students feel the subject mathematics difficult [3, 4, 5]. One of the reasons behind this is that the students are unaware of its application and its importance. We feel that the theory taut in the classes and its application to solve real time problem must go hand in hand. Our aim is to introduce the reader with application of mathematics in various subjects like physical sciences, chemical science, life science, social science etc. We also put a light on recent advancements in these topics, which are solved by the mathematical tools.

We strongly feel that this will introduce the reader with real time application of mathematics. This will help in creating interest among students and also reducing the unnecessary fear about the mathematics. This article will also help the researcher to find many real world problems to work on.

Chapter is arranged in five sections. First is the introduction. In section two, we see the application of mathematics in networking. Section three deals with mathematics in plants studies. In fourth section, we discuss mathematics involved in the chemical sciences. Last section deals with the physical phenomenon applying mathematics.

Networking:

1. What is networking?

A network is any collection of computers, servers, mainframes, many network devices mobiles, tablets, switches etc. those are connected by links through which these devices can share the data. Internet network, Local area network (LAN), Wide area network (WAN), Personal area network (PAN) and many social networks introduced by Smartphone, banking networks are few examples of the network.

2. Importance of network:

The network has wide range of applications due to many of its useful advantages [6]. We list few of them here. With the help of the network, one can share any data and information on each of the device connected through the network. Moreover, it allows an access to common database, which help to reduce the cost of the project and promote he collaborative work. It provides the user different modes of communication like phone, email, chatting and conferencing of call etc. With the help of network, one can share the hardware to al the user connected through the network. For example, all users can access the single printer which is on the network. All users can access the supercomputer to speed up their tasks. All users in network can also share much software.

This helps in enhancing the performance of each device in the network with very less amount of expenditure. Network also eases our day-to-day life. For example, we can perform many financial transactions if we are the part of banking network.

3. Network topology:

A network topology is the physical arrangements of the network devices joined by the physical links. Different types of network can be seen depending upon this arrangement. Ring, bus, linear bus, star, tree, mesh, torus are few examples of basic network topologies that have particular arrangement of devises.

Parallel processing is the process in which different components in the network work simultaneously on different issues. This achieves faster execution time and it enhances the processing capacity.

In recent years, the interconnection network is considered as an important choice for architecture of parallel systems in future. The base of this choice is to determine an efficient network for the communication. The computer is itself an interconnection network as it form by millions of processors that facilitates the data transfer without any congestion. Thus in the modern edge the designing of efficient topological networks is emerging as a highly significant problem. Big networks are difficult for handling and for maintaining. Thus, it is require developing methods for the study of such a network. The method should be efficient, easily scalable and reduce the financial burden of building and maintaining the network. Graph theory a branch of the mathematics provides a theoretical tool, which serves all this requirements. It provides a theoretical way to model the network.

4. Mathematical modeling of a network:

The study of the network can be done with help of underline graph structure. Network can be modeled into the graph in which vertices represent the network devices and edges represent the communication links. Graphical model of the network provides a simplest way to study complex networks. Many important properties of the network can be studied with the help of the graphical model.

We now recall the basic definitions from graph theory, which can be, found in any standard book of graph theory and discuss various parameters of the graph that affect the performance of the network.

4.1 Graph

By a set we mean any collection of well define objects. An undirected *G* is a pair of two sets *V* and *E* that is *(V, E)* such that elements in *E* are unordered pairs of elements in *V*. Elements in *V* are called as vertices of *G*, while elements in E are edges of G. Many times, we use notation *V(G)* and *E(G)* for the sets *V* and *E*, respectively. Each element *e* in *E* joins two elements *u, v* in *V*. Edge joining two vertices *u* and *v* is also denoted as *uv* or *(u,v).* If *e=uv,* then we call *u, v* as end vertices of *e.* We also say this as an edge *e* incident on vertex *u* and *v*. The order of *G* is denoted by |*V(G)*| and it is same as number of elements in the set *u, v* If |*V(G)*|=*n*, then we call *G* as an *n*-vertex graph. If *n* is finite, then *G* is a finite graph. Cycle, path, hypercube, mesh are some popular graphs used in the networking. Following are the basic properties of the graph.

- Total number of edges incident on a vertex *v* is called as degree of *v*. If each vertex of the graph has same degree *k*, then we call it as a *k*-regular graph.
- The diameter of a graph is the largest distance between any two vertices of the graph.
- The girth of the graph is length of the shortest cycle in the graph.
- If there is an edge between any two vertices of a graph, then it is known as a complete graph. A complete graph on n vertices is denoted by $K_{n.}$
- If there is a path between any two vertices of G, then G is connected. The connectivity of G, denoted by $\mathrm{K}(\mathrm{G})$ is the minimum cardinality of a set S of vertices of G such that $G-S is either disconnected or K_1. The edge-connectivity, denoted by $\lambda(G)$ is defined similarly. If $\mathrm{K}(\mathrm{G}) \geq \mathrm{n} \geq 1$, then *G* is *n*-connected. Similarly, if $\lambda(G) \geq n \geq 1$, then *G* is *n*-edge connected. A maximal connected subgraph of *G* is a component of *G*.
- Connectivity of the underline graph plays an important tool to measure fault tolerant capacity of the network.

- Graph H is a subgraph of G if all the vertices of H are the vertices of G and each edge of H is also an edge of G. A spanning subgraph is a subgraph containing all the vertices of G.
- A graph is said to be pancyclic if contains the cycles of every length l such that $3 \leq l \leq |V(G)|$. We call it as bipancyclic if we restrict l to the even integer.

The extensive applications and study of the interconnection network identified a list of graph theoretic various properties that architecture of the network should have. Such properties are well studied in the literature. We discuss few of them here.

Few characteristics of a good network:

Following are the few characteristics using which one can decide the practical applicability of the network. These characteristics are well studied for the popular network like hypercube, hypercube variants, torus etc. [7,8, 9]

- Network with a small diameter reduces the cost of communication.
- Network is fault tolerant if it remains functional along with few faults if processors or links. Fault tolerance capacity of the network can be studied with the help of connectivity of the underline graph. A network is stronger if it has higher the connectivity.. Traditional connectivity defined in the above paragraph some limitations to measure fault tolerance capacity of the network. Hence, various new version of the connectivity are come out. Harary [10] introduced the concept of conditional connectivity of a graph to overcome such limitations of traditional connectivity. Finding a network with high conditional connectivity is an important problem to study. Recently this problem acquires the attention of many researchers. Conditional connectivity of many well-known interconnection networks is determined.
- A subgraph H is said to be embeddable in a graph G if G has a subgraph isomorphic to the graph G. Good embedding property of the network allows us to run different algorithms simultaneously. Cycle embedding capacity is equivalent to pancyclicity or bipancyclicity of the underline graph. The various extensions of these concepts such as vertex pancyclicity, edge pancyclicity or geodesic pancyclicity of various networks are new areas to work on.
- One can experiment the algorithm on a network in low cost if it is tried initially on some small scale network. This small scale network should be as efficient as original network and retain desire properties of the parent network. Finding such a small scale network is equivalent to finding subgraphs of underline network of the graph having desire properties. Hence a subgraph of graph with desire properties is an important problem to study.

- Network should be scalable that is one can add or remove desire number of vertices or edges by retaining required properties of the network.
- Recursive nature of the underlined graph increases the scalability of the network. Recursive nature of a graph also helps in studying the network. In such a network one can observe and easily generalize various properties of smaller network inductively to the bigger network.
- Every vertex in the graphical model of a network corresponds to a processor and has a unique address. That is it has distinct characteristics. Hence, it is better if we could label each and every vertex of the underling graph. The aim of the labeling scheme is to reduce communication gap. Recursive nature of the networks helps to find out labeling scheme of the network. In past few years, labeling of the graph is coming out to be a new branch of the mathematics.
- Routing is an important issue for the network. Routing is the process of selecting a right path for the traffic for smooth, noise free and faster communication. Degree of vertex gives us a number of path available at that vertex through which message can be passed. Algorithm for selection of right path at each level is the routing algorithm. Knowing the shortest path between any two vertices of the graph helps in designing routing algorithm. Graph theory provides different algorithms to determine shortest path between any two vertices of the graph.
- Another important issue in the network analysis is the broadcasting. A processor can send the message to all its neighbors. Each receiving node must pass on the message to its neighbors. In the similar fashion message goes in lower levels of the network. In this procedure, unnecessarily a node can receive same message more than once. This can be avoided by transforming the message along tree. This leads to designing broadcasting algorithm. Thus, study of trees in a network is directly related broadcasting of the network.

Apart from these, many more properties of underline graph such as average degree, Minimum and maximum degree, decomposition properties gives us important properties of the network. Looking all in the consideration one can choose the proper graph structure for the architecture of the network. The network for which underline graph is rich in the property can be chosen for the practical implementation. Hypercube, Mesh, torus, Peterson graphs [11,12] are few such example of graph used for the construction of the network.

Mathematics in the study of plants:

In this section we are going to see how mathematics is useful for measuring the strength of the plants. We introduce the reader with few models and review various factors regarding the strength analysis of the plants.

Motivation for stress analysis of the plants:

The plants have an enormous importance in day today life of human beings. A specifically agricultural crops plant was utilize for the welfare of human beings. Plants are well adopted with nature and can manage to balance their own weight. But extreme weather conditions create stress on tree branches which leads to their damage. Extreme weather events are expected to increase worldwide, therefore, their effects on crop yields is important for topics ranging from food security to the economic viability of biomass products.

In the agricultural sector, destroyed crops can be a major source of losses of floods and storm in the short-term. Necessary purchase of fodder and the dependence on imports might generate additional losses in the long run.

The Indian economy totally depends on agriculture. In Indian agriculture different types of plants grown which includes fruit plants, cereal crop plants, legumes crop plants etc. This agriculture production is totally depends on the weather conditions. Extreme weather events are having significant impact on agriculture and food security which is the main source of income to a large section of the rural population in our country.

The growing incidence and severity offloads, hailstorms and other extreme weather events severely affect the livelihood options for small-scale farmers. It is disturbing to note that India loses, annually, about 2% of its GDP and around 12 percent of central government revenues to natural disasters (IPCC). To overcome this problem is having immense importance considering different aspects. We could not stop such weather situation but improve crop plant strength through breeding or providing necessary support to it that can avoid maximum damage.

In crop plant due to uncertain weather condition includes storm and flood drastically affect on yield of crops (From the meteorological point of view, storms are low-pressure areas. In this paper, the term 'storm' is used in the narrower sense for heavy winds). It is directly affects on socioeconomic status of farmers. The metrological department predict future weather condition but unfortunately farmer did not having any solutions to overcome this situation.

Thus the problem development of systematic methods attracted the attention of many researchers. From such models we can given or predict the type of support necessary to withstand in uncertain weather condition. The study on mechanical tissue and their patterning in different plants and its relationship to strength ability helps breeder to develop a breed that can withstand during uncertain weather condition.

Role of mathematics in stress analysis of the plant:

Necessity of stress analysis:

The mathematics gives a theoretical model to calculate the strength. We can apply this model in the field of plant science to measure the strength of crop plant or its branches. We also provide artificial mechanical, airflow and water flow conditions to estimate the strength because the natural environmental condition like storm and flood damage crop mostly. Beside this to find the better crop variety which having good strength we will evaluating different crop varieties. Therefore, mathematics influences plant anatomy, plant breeding, plant genetics, crop science and environmental science.

Mathematics behind the stress analysis models:

Study of mechanics of materials plays an important role in determine strength of physical performance of various structures like building, bridges and road. Strength can be defined as the capacity of the object to support loads. To avoid structural failure, one must consider the actual load capacity of the structures. This theory is well developed and various ready-made tools and machines are available to measure strength of these structures [13].

Similar study for plants will be beneficial to understand stress on various parts of plant. This would be useful to determine sustainability of that plant. This also helps to study tree morphology [14]. Analysis of mechanics of stress on trees remains an attraction for many researchers in last few years [15 - 21]. Many mathematical models were developed to stress analysis in tree [22].

Evans *et al.* [23] calculated tree branch stress by modeling them as tapered cantilever beams with circular or elliptical cross section area and with uniform modulus of elasticity [23]. In [19] important property modulus of elasticity in plants was discussed. In [24] stress analysis of tree branches is done by considering radial variance in modulus of elasticity. This study also considered curviness of branches. In [24] they have obtained formulae for axial compressive stress and searing distributed stress was given. Most of the previous study was done for the tree or braches in which stress is only due to weight of the branch is considered. Nevertheless, for the crop plant many other factors such as wind flow and water flow need to be consider other than weight of the branches. Burgess S. C. [17] et al. studied bending stress due to wind is under some special condition. In our study, we are formulating stress analysis in crop plant under heavy wind and water flow.

Methodology used in various models:

1. Allowable load is the highest load that plant can bear. That is highest load which does not cause any permanent damage in the plant.
2. Various weights will be putted on different parts of tree to calculate allowable load.
3. This process of loading and unloading can be repeated for successively higher values of load.
4. Eventually, a stress will be reached to a point, which causes permanent damage to our plant.
5. By this procedure, it is possible to determine an upper limit of the load that is allowable load.

6. As describe in [9] Allowable load = Allowable stress*(Area). Using this formula we can calculate allowable stress for that particular plant. We will denote this by A_s.
7. As mention in [5] bending movement is occurred due to component of weight acting perpendicular
8. Our plant remain undamaged until Normal stress = Allowable stress

   That is $A_s = \frac{mg\ sin\theta}{L}$ where θ is the angle of deflection for vertical axis of plant is and is the length of the plant.
9. For above formula $A_sL = mg\ sin\theta$ that is $sin\theta = \frac{A_sL}{mg}$. For which one can easily calculate angle θ.

   This angle is known as allowable deflection denoted asθ_a. This is the maximum deflection, with which our plant can sustain.

Mathematics in Chemical sciences:

Mathematics is a tool to simplify complex natural phenomena into easily perceivable human understanding. From the very beginnings of chemistry, mathematics was used to create quantitative and qualitative models for helping comprehend the world of chemistry by understanding the elements that make up molecules. An atom is made up of particles, which are known as protons, neutrons, and electrons. Measurement issues concerning these particles are a big part of what chemistry is about. Protons, neutrons, and electrons have mass & electrical charge, mass and charge can be measured. Patterns in the mass and charge of atomic particles helped chemists get insight into the nature of atoms and the molecules these atoms can form.

Mathematics helps to chemist to measure the value of rate of reactions, rate constants, and molecular energies in chemical kinetics. The density functional theory now days widely used to study molecular dynamics, which helped us to dipper in the molecular behavior. Derivative, Integration & Fourier transforms are the few useful mathematical tools widely used for chemical analysis [25-27].

Algebra in chemistry:

One interaction of mathematics and chemistry that is so familiar now, is the use of numbers in balancing a chemical reaction. In a chemical reaction, the masses of the inputs to the reaction must be the same as the masses of the products produced by the reaction. A good example of the complex ways mathematics helps one-grasp chemistry issues is the recent concern with rising levels of carbon dioxide in the atmosphere, and the pollution that comes from using coal as a source of energy. One way to deal with the issue of the negative aspects of burning coal is to use more natural gas, whenever possible, as a source of energy. One of the major components of natural gas is methane. Methane is an example of a *hydrocarbon,* a molecule made up of hydrogen and carbon atoms. Methane when ignited undergoes a chemical reaction that releases energy. The energy is in part stored in the bonds that keep the molecule from separating into the components that make up methane - the hydrogen and carbon atoms. In Figure 1, if one knows that methane and oxygen mix to produce carbon dioxide and water, the question is how much of each of the "reagents" are involved? **Where the coefficient do numbers shown for the chemical reaction below, the 1, 2, 1, 2 respectively, come from?**

$$CH_4 \quad + \quad 2O_2 \longrightarrow CO_2 \quad + \quad 2H_2O$$

Figure 1: Balancing equations of molecular reactions

It turns out that one can use ideas from the **algebra** learned in high school (or in a linear algebra class) to use systems of linear equations to "balance" chemical reactions in this spirit. The idea is to introduce letters for the molecules involved and then to use chemical principles for producing these equations. For example, in the equation above we have four molecules CH_4, O_2, CO_2, and H_2O. Suppose we use the letters x, y, z, w for the numbers of these molecules, respectively. Can we deduce relationships between the values of x, y, z and w? Since C appears on the left and right of the "equation" (bottom of Figure 1) we can say that $x = z$. What about oxygen? We have $2y = 2z + w$ because there are two oxygen atoms on the left, and two atoms in the carbon dioxide and one in the water. Finally for H, we have $4x = 2w$. Notice here we have more "unknowns" than equations and if there is a way to balance the reaction we will need to use nonnegative integers as the values for x, y, and z. Why not try a solution where we find the unknowns, y, z, and w in terms of x? Using some simple algebra, we see that:

$$y=2x, \qquad z=x, \qquad w=2x$$

The smallest value x can have is 1. What does this mean for the other values? if $x = 1$, then y must be 2, z must be 1, and w must be 2. Thus, we get exactly the values for the coefficients of CH_4, O_2, CO_2, and H_2O we see in Figure 1. In some cases the linear equations that we get have no positive integer solutions. Either by predicting the amounts of reagents in a chemical reaction or showing that the reagents can't combine in a "legal" way, linear algebra is a tool for insight.

Graph theory in chemistry:

The value of graph theory to chemistry started to become apparent in the 19th century. Work by two British mathematicians, Arthur Cayley (1821-1895) and James Joseph Sylvester (1814 -1897), laid the ground for a long tradition of successful use of graph-theoretical ideas in chemistry. From early on chemists have used diagrams to help them think through issues involving molecules. For example, here is the way a methane molecule might be drawn.

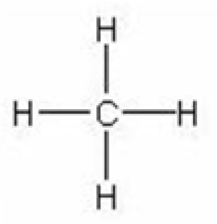


Figure 2: Diagram of methane molecule

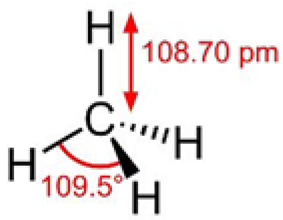


Figure 3: Tetrahedral geometry

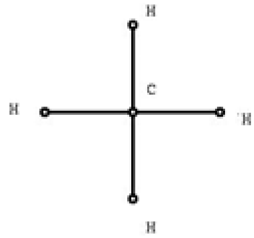


Figure 4: Graph representing methane

Other diagrams might give more of a sense that the methane molecule is three-dimensional, and give a more accurate feel for the molecule because the labels give information such as bond length or angles between bonds. In Figure 3 the drawing is designed to show the "tetrahedral" nature of the methane molecule, which is harder to see in Figure 2. One of the things mathematics typically does is formalize "ad hoc" tools and use the general theory that evolves from the mathematical approach to advantage. Diagrams such as the one chemists might draw can be considered in the framework of geometrical diagrams called *graphs*. Graph theory involves studying the properties of diagrams that make use of dots and line segments. The dots of a graph are called *vertices* and the line segments are called *edges*. When chemists use these diagrams, they are usually called ***molecular graphs* or *structural formulas***. A structural formula for methane is shown in Figure 4. The dots represent atoms, labelled to show what kind of atom is involved, and the edges represent bonds formed between the atoms. Different authors use the same or similar terms in different ways. Here the diagram does not have metric (distance) information, and the angles between bonds and the lengths of bonds is not part of the information that the diagram is conveying. Therefore, graph theory is a kind of geometry but geometry that does not use "metrical" information directly.

Use of differentiation in chemistry:

The concept of derivative or differentiation is used in chemical kinetics to find out rate constant, rate of reactions. Chemical kinetics is the branch of Chemistry which deals with study of rate of reactions & mechanism of the reactions. Rate is change of physical quantity per unit time.

eg. Velocity = Distance / Time,

Acceleration = Velocity / Time etc.

The rate of reaction is defined as the decrease in concentration of a reactant per unit time or increase in concentration of a product per unit time. The variation of concentration of reactants with time is depicted in figure 7. The concentration of reactants decreases with time and concentration of product increases with time.

Consider simplest reaction,

	A	→	B
	Reactant		Product
When time, t=0	$[A]_0$		0
When time, t = t	[A]		[B]

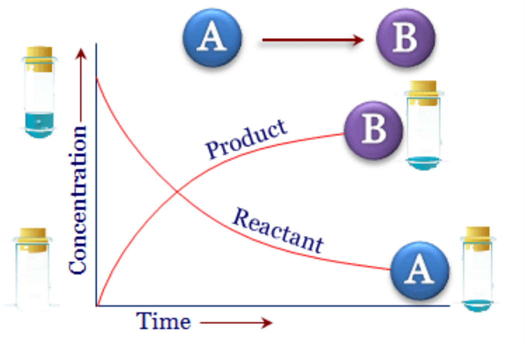


Figure 5: Variation of rate of reaction with time

Rate of reaction = rate of disappearance of reactant A

= rate of appearance of product B

∴ Rate of reaction = rate of disappearance of A = $-\frac{d[A]}{dt}$ …….1

Or

∴ Rate of reaction = rate of appearance of B = $\frac{d[B]}{dt}$ ……...2

Where,

[A] & [B] are the concentration of the reactant & product respectively

Negative (-ve) sign shows that concentration of reactant decreases with time

At given temperature, the rate of a chemical reaction at given time is directly proportional to the molar concentration of the reacting substance at that time, therefore, equation 1 becomes,

∴ Rate $= -\frac{d[A]}{dt} \propto [A]$ ………..3

Equation 3 represents that rate of a chemical reaction is dependent on the concentration of the reactant; hence, this equation is known as rate equation or rate law.

∴ Rate $= -\frac{d[A]}{dt} = k_1 [A]$ ……..4

where k_1 = First order rate constant

∴ $-\frac{d[A]}{dt} = k_1 [A]$

Integrating above equation between the limits,

[A] to $[A]_0$, we get,

$\therefore \int_{[A]0}^{[A]} - \frac{d\,[A]}{d\,t} = \int_{t=0}^{t=t} k_1\, dt$

$\therefore \ \ln \frac{[A]_0}{[A]} = k_1\, t$

$\therefore \ \ k_1 = \frac{1}{t} \ln \frac{[A]_0}{[A]}$ ……….5

Equation 5 is the expression for first order rate constant.

Let, initial concentration of reactant , $[A]_0 = a$

Concentration of product at time t = x

Concentration of unreacted reactant = [A] = a – x

$\ln c = 2.303 \log_{10} c$

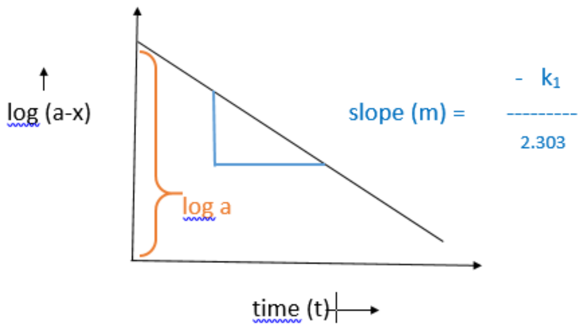
$\therefore \ \ k_1 = \frac{2.303}{t} \log \frac{a}{a-x}$ ……….6

$\therefore \ \log a - \log (a\text{-}x) = \frac{k_1 t}{2.303}$

$\therefore \ \log (a\text{-}x) = - \frac{k_1}{2.303}\, t + \log a$ ………7

Equation 7 is similar to straight line equation y = mx + c.

Plotting log (a-x) verses time 't', we get straight line equation as shown below,



Hence we can conclude that mathematics supports chemists to simplify molecular understanding, physical and chemical behaviour of molecules in better way.

Mathematics in physical sciences:

Application of Conformal Geometry and Ultimate Fate of the Universe

Conformal geometry is associated with angle preserving transformation. The birth of the universe is still unexplainable, but the satisfactory explanation is given by the cosmological model put forward by Sir Roger Penrose named conformal cyclic cosmology. This theory claims the birth of the universe is a manifestation of the death of the previous universe and it is an endless infinite cycle of death and birth of the universe [28].

The mathematical model used to build this theory is conformal geometry and general theory of relativity [29]. As Newton's gravitational theory is inadequate and imperfect and wrong while applying to the whole cosmos as a one unit. This is only after Albert Einstein who proposed the general theory of relativity. This theory is extremely powerful and universal. The GR gives tensor equation given by

$$R_{uv} - \frac{1}{2} R g_{uv} + \Lambda g_{uv} = \frac{8\pi G}{c^4} T_{uv}$$

The above equation leads to about 10 field equations and the left side deals with the curvature of space and time while the right side of equations is mass energy of the universe. This equation simply tells mass and energy curves the space and curved space change the energy of mass. From above equation, Friedman derived the equation for whole universe to predict its fate by considering [which are true even today]

$$(\frac{\dot{a}}{a})^2 - \frac{8\pi G\rho}{3} - \frac{\Lambda c^2}{3} = -\frac{kc^2}{a^2}$$

In above equation the Λ is cosmological constant and associated with anti-gravity effect, the universe is expanding but its expansions rate is increasing gradually that means there is some unknown energy i.e. dark energy stretching in accelerated manner the fabric of space and time. It seems to be dark energy is the property of vacant space and this dark energy increasing gradually in whole universe [28].

Aon is a time gap between two successive birth and death of universe and in between Aons for universe conformal rescaling has no physical significance and universe has sense of length. There is clearly a difference between one light second and one light year. As even if the universe has a single rest mass particle then the universe has a sense of time and so the sense of length and conformal rescaling is invalid. In short the sense of scaling is because rest mass particle build a clock [29, 30].

In the far future, the universe will stretch infinitely thanks to dark energy and all matter, star stellar remnants will decay into mass less photons, gravitons, and elementary particles.

Still universe has Black holes, they carry mass and so still universe has sense of time. But it is very clear that all black holes decays into photons by Hawking's radiation [31].

$$T_H = \frac{hc^3}{16\pi^2 G k_B M}$$

Hawking's radiation evaporate black holes extremely gradually but at end there are no back holes at all Even black holes will decay by Hawking radiation into photons. Still universe has the elementary particles and still universe has sense of time. The mass of elementary particle is because of the Higgs field given by Higgs equation [32].

a $(\text{mass})^2 \equiv (M_H)^2 = 2|\langle\mu\rangle|^2$

But in the far future because of extreme expansion the Higgs field will decay into lower energy and so will not grant mass to any particle [33]. In the far future the universe has infinitely stretched space time and massless particles and so clocks and so no sense of length. That late universe lost its sense of scaling.

What about Early universe and how exactly late dying universe and ne born early universe connected? According to conformal cyclic cosmology infinitely, stretched late dying universe is the same as that of a zero sized very early just born universe on valid conformal transformation. Early universe was made up of photons and elementary particles, now the question arises do elementary particles have mass at that time? The temperature of that universe was very high and at that temperature the Higgs field did not grant mass to them. In short early universe also had all massless particles and son that early universe had no sense of time and so no sense of the length.

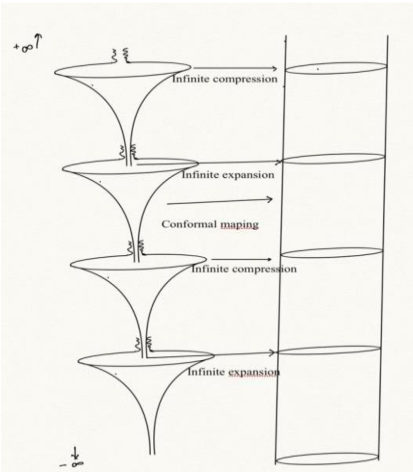


Figure 6: Schematic representation of conformal mapping

Under conformal rescaling, the end of the late universe is the birth of new universes in both cases with no sense of scaling. Therefore, there are infinite cycles of births and deaths of the universe called as cyclic universe [34].

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About Editors

Dr. Med Ram Verma is presently working as a Principal Scientist at ICAR-Indian Veterinary Research Institute Izatnagar, Bareilly, Uttar Pradesh. He passed M.Sc. in Agriculture Statistics in 1998 from Dr. Y.S. Parmar University of Horticulture and Forestry, Nauni, Solan, Himachal Pradesh. He obtained his Ph.D. degree in Statistics in 2005 from Dr. B.R. Ambedkar University, Agra, Uttar Pradesh. He joined Agricultural Research Service in 2003. He guided 4 Ph.D. and 10 M.V.Sc students in the discipline of Biostatistics. He published 227 research papers in reputed journals. He authored/Edited 5 books. He was awarded with Best Teacher Award for the year 2014-15 by ICAR-IVRI Deemed University, Izatnagar, Bareilly, U.P. Dr. Med Ram Verma was awarded with prestigious Bharat Ratna Dr. C. Subramaniam Award for Outstanding Teachers 2018 by Indian Council of Agricultural Research. He is the Fellow of Royal Statistical Society, London (UK) and Elected Member of International Statistical Institute, Netherlands. He is the Editorial Board Member and reviewer of the several journals.

Dr. Megha Abhiman Bhamare (Patil) holds the degree of M. Sc. (Mathematics), M.Phil. & Ph. D. She is presently working as Head of the Department of Mathematics in Krantiveer Vasantrao Narayanrao Naik Arts, Commerce and Science College, Nashik. She has 22 years of experience of teaching Mathematics. She has also worked as paper setter and moderator. She has published number of papers in national and international journals.

Dr. Sheetal Ramdas Gomkar is presently working as Assistant Professor and Head, Department of Mathematics, Janata Mahavidyalaya, Chandrapur (M.S.) 442401. She did her Ph.D. in the year 2014 from R.T.M. Nagpur University, Nagpur, India. Dr. Gomkar has 13 years of research experiance. The main area of research is special Relativity. Dr. Gomkar published 13 research papers in various journals and proceedings and two books published in various national level publication. He attended and presented research papers in various national and international conferences.

Dr. Hariom Singh Tomar is presently working as Assistant Professor and Head at Department of Mathematics, Chintamani College of Arts and Science, Gondpipri Dist. Chandrpur (MS) from 2013. He previously worked as a Guest Lecturer in the Department of Mathematics, Agra College, Agra to teach B. Sc. from July 2004 to 2010 and worked as a Contact Lecturer in the Department of Mathematics, G.N.N.(P.G.) Iradatnagar, Agra teach B.Sc. from July 2010 to 2013 .He has completed M. Phil in Mathematics from Dr. B. R. Ambedkar University, Agra in 2001 and completed Ph.D in Mathematics from Dr. B. R. Ambedkar University, Agra in 2010. Dr. Tomar has published 13 research papers, 21 Books and 4 chapters in various international and national Journals and Publishers. He has attended 50 international and national conferences,20 workshops and 6 Faculty Development Programmes. He is serving as reviewers for various reputed journals like International journal of Mathematical Sciences.

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