RESEARCH ARTICLE

STUDY ON MULTI-OBJECTIVE MULTI-LEVEL DECISION-MAKING SUPPLY CHAIN MANAGEMENT UNDER FUZZY OPTIMIZATION THEORY

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Abstract: In order to handle uncertainty in supply chain networks that make decisions at multiple levels and with multiple objectives, this study introduces a fuzzy-based optimization method. To solvethe multi-objective optimization problem under uncertainty, the suggested method integrates fuzzy set theory with mathematical programming. The uncertainty in demand, supply and transportation is represented using fuzzy numbers and the fuzzy optimization model is solved using a fuzzy programming algorithm. The results demonstrate the effectiveness of the proposedapproach in optimizing uncertainty in multi-objective multi-level decision-making supply chainnetworks.

Keywords: Multi-objective; multi-level decision-making supply chain network; uncertainty optimization; fuzzy approach; fuzzy programming.

Introduction:

In today's global competition, various products must be produced according to customer's demand. There is also an increasing trend for high-quality products, and many firms prefer to outsource part of their needs. In today's competitive market, enterprises and manufacturers, in addition to considering organization and internal resources, need to manage and control resources and related elements outside of the organization because they want to achieve a competitive advantage or advantages with the purpose of gaining more market share. Accordingly, activities such as supply and demand planning, material procurement, production and product planning, inventory control, distribution, delivery, and customer service executed at the company level. Controlling and coordinating all activities play an essential role in supply chain management. Supply chain management (SCM) is a phenomenon that does this in a way so that customers can receive fast and reliable services and quality products at reasonable expenses. In this regard, optimization of the supply chain has become a very important issue for many companies. Supply chain optimization is the application of processes and tools to ensure the optimal operation of a manufacturing and distribution supply chain. This includes the optimal placement of inventory within the supply chain and minimizing operating costs (including manufacturing costs, transportation costs, and distribution costs). Implementation of appropriate strategies for minimizing costs and increasing flexibility in a competitive and complex market is one of the challenges for supply chain optimization.

On the other hand, many planning problems require the synthesis of decisions from several interacting individuals or agencies. Often, these groups are arranged within a hierarchical administrative structure, each with independent and perhaps conflicting objectives. Multi-level mathematical programming (MLP) is identified as mathematical programming that solves decentralized planning problems with multiple executors in a multi-level or hierarchical organization.

In many real-world supply chain optimization problems, we are faced with multiple objectives, which are also often in conflict. Traditionally, most studies have been focused on revenue maximization or cost minimization as a single-objective optimization problem. However, in a real supply chain, managers may be looking to optimize multiple conflicting objectives, such as reducing costs and increasing customer satisfaction simultaneously.

In addition, uncertainty is the main factor that influences the efficiency and coordination of the supply chain and tends to spread up and down the chain, significantly affecting its performance. This uncertainty may occur in various instances, such as uncertainties in demand, supply (delivery time, etc.), costs, and supply chain structure, and so on.

The supply chain optimization may be considered at different levels, depending on the strategic, tactical, and operational variables involved in decision-making (Mele *et al.*, 2007). The strategic level concerns those decisions that would have a long-lasting effect on the firm, such as the size and location of production, warehouse and distribution departments, technology selection process, production, etc.

- ***SUPPLY CHAIN programming***

The standard mathematical programming problem deals with finding an optimal solution for just one decision maker. Nevertheless, many planning problems contain a hierarchical decision structure, each with independent and often conflicting objectives. These types of problems can be modeled using a multi-level mathematical programming technique. A bi-level programming problem, which is a special case of a multiple-level programming problem, involves two optimization problems where the feasible region of the upper-level problem is determined implicitly by the solution set of the lower-level problem. The decision maker at the upper level (the leader) optimizes his/her objective function independently and is influenced by the reaction of the decision maker at the lower level (the follower), who makes his/her decision after the leader. Briefly, each decision maker independently pursues his/her own interest but is influenced by the action of the other decision maker. This hierarchical decision process arises in many fields, including decentralized resource planning, highway pricing, the power market, logistics, economics, manufacturing, and road network management, which are referred to as multilevel decision problems (Zhang *et al.*, 2010). It has been proved that solving the bi-level linear programming is an NP-hard problem, and even finding a local optimal solution of the bi-level linear programming is NP-hard (Wang *et al.*, 2010). A linear bi-level programming problem has an important property: at least one global optimal solution is attained at an extreme point of the constraint region (Gao *et al.*, 2010). Based on this property, many algorithms have been proposed for solving bi-level linear programming problems (Shi *et al.*, 2005-a; Shi *et al.*, 2005-b; Gao *et al.*, 2010; Zhang *et al.*, 2010; Zheng *et al.*, 2011). These algorithms can be roughly classified into three categories:

1. **Vertex enumeration-based approaches** (Zhang *et al.,* 2010; Ansari & Zhiani Rezai, 2011; Calvete & Gale, 2012),
2. **Kuhn-Tucker approaches** (Calvete & Gale, 2004; Lu *et al.*, 2006; Shi *et al.*, 2005-b; Mishra *et al.,* 2007), where a bi-level programming problem is transformed into a single-level problem by including the follower's optimality conditions as extra constraints, and
3. **Heuristics** (Gao & Liu, 2005; Wang *et al.,* 2008; Kuo & Huang, 2009; Lin *et al.*, 2008; Jiang *et al.*, 2013), which are global optimization techniques based on convergence analysis.

The Kuhn-Tucker method is the best-known approach for solving bi-level programming problems. The main strategy of the Kuhn-Tucker approach is to replace the follower's problem with its Kuhn-Tucker conditions and add them as constraints to the leader's problem. Reformulating a linear bi-level programming problem into a standard mathematical program makes it relatively easier to solve, as all constraints are linear. With the elimination or reduction of these constraints, a standard linear programming problem is obtained, which can be solved using the simplex algorithm.

In a multi-level decision-making problem, decision makers at one level may consider multiple objectives simultaneously, and these objectives are often in conflict. Many papers have addressed bi-level single-objective programming problems, but fewer studies tackle bi-level multi-objective problems (Zheng *et al.*, 2011). Osman *et al.* (2004) presented an approach using fuzzy set theory for solving bi-level and multiple-level multi-objective problems. Baky (2010) studied a fuzzy goal programming (FGP) algorithm for solving decentralized bi-level multi-objective programming problems. The fuzzy goals of the objectives are determined by identifying individual optimal solutions. These fuzzy goals are then characterized by associated membership functions, which are transformed into fuzzy flexible membership goals by introducing over- and under-deviational variables and assigning the highest membership value (unity) as the aspiration level to each of them. To elicit the membership functions of the decision vectors controlled by the upper-level decision maker (DM), the optimal solution of the upper-level multi-objective linear programming (MOLP) problem is separately determined.

The original bi-level programming technique mainly deals with one leader and one follower decision problem. In real-world applications, multiple followers (i.e., multiple decision units at the lower level) may be involved. Thus, the leader's decision will be affected not only by the individual reactions of those followers but also by the relationships among them. For each possible solution of the leader, these followers may have different interactions. These followers may or may not share their decision variables. They may have individual objectives and constraints but work cooperatively with others, or they may have common objectives or common constraints (Lu *et al.,* 2006).

Shi *et al.* (2007) proposed an extended *K*-th-best approach for linear bi-level multi-follower programming problems with partially shared variables among followers. Wang *et al.* (2009) studied a class of bi-level multi-follower programming in which there are partially shared variables among followers. A fuzzy interactive decision-making approach was proposed to derive a satisfactory solution for decision makers, considering not only the dominant action of the leader but also the ratios of satisfaction between the leader and the followers.

Zhang and Lu (2010) considered a multi-objective multi-follower linear bi-level programming problem with fuzzy uncertainty in parameters and a cooperative relationship between followers. They solved the problem using the *K*-th-best method. Among other studies in this area, Ansari and Zhiani Rezai (2011) examined a multi-follower problem with an uncooperative relationship.

Paper Organization:

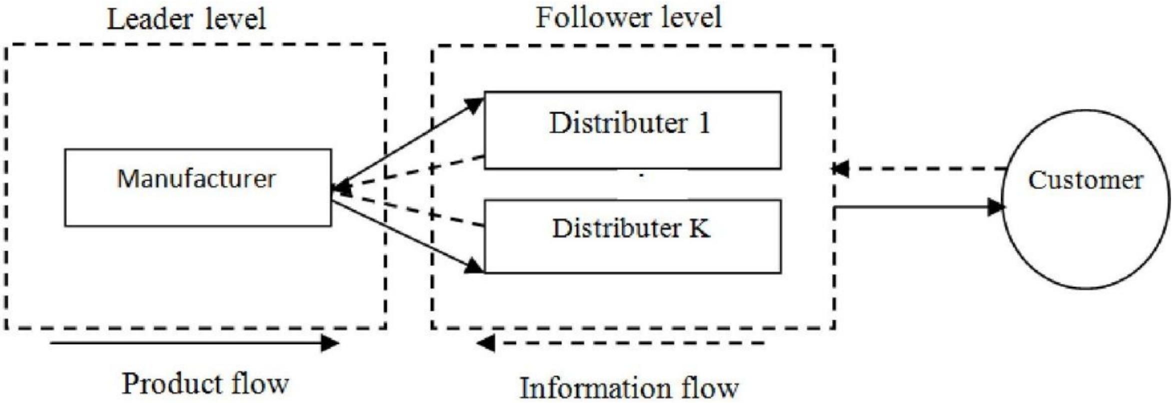
This paper is structured as follows:

1. **Model formulation** is presented first.
2. **The solution procedure** is described in Section 3.
3. **An illustrative example** of a supply chain with one manufacturer (first level) and two distribution centers (second level) is provided in Section 4.
4. **Discussion, concluding remarks, and future research directions** are summarized in Section 5.

Supply chain management model

Problem description, assumptions, and notations

The bi-level problem with multi-products, multi-objective and multi-follower in supply chains are examined here. Consider a supply chain with two levels with a manufacturer as a leader at high level and K distribution centers as K followers in low level. Manufacturer has been hegemonic power in the chain and distribution centers as followers should adopt the best decisions with regard to decisions of manufacturer. The manufacturer has produce M products and distributed them to distribution centers that sale products in a same market. Assume that the market demand and costs is normally fuzzy/imprecise due to incomplete and/or unobtainable information. Fig.1 illustrates the supply chain structure.



The fuzzy mathematical programming model designed here is based on the following assumptions:

1. Demand and cost functions are fuzzy with imprecise as privation levels.
2. All objective functions and constraints are line are equations.
3. The production costs and distribution cost/time at manufacturer level and distribution cost/time on a distribution centers are directly proportional to the units manufactured and shipped capacity per truck, respectively.
4. The pattern of triangular distribution is adopted to represent all of the fuzzy / imprecise numbers and the linear membership functions are specified for all of the fuzzy numbers involved in the proposed model.
5. Shortages not allowed in any of levels.
6. Each distribution center has determined minimum inventory level.
7. There is no collaboration between followers.

1. Fuzzy theory:

In this section we recall some basic definitions and arithmetic operations.

Definition: If X is a universe of discourse and x be any particular element of X. The fuzzy set B defined on X is a collection of ordered pairs,

$$P = \{(x, \mu_P(x)) | x \in X\}$$

Where, $\mu_P(x): X \to [0,1]$ is called the membership function. i.e. $0 \le \mu_P(x) \le 1$

Definition: Fuzzy set B is defined on a set R of real number whose membership function $P: R \to [0,1]$ is a fuzzy number under certain conditions:

$P: R \to [0,1]$is normal. (i.e. height $(P) = 1$)

$P: R \to [0,1]$ is convex

$P: R \to [0,1]$is piecewise continuous.

Definition: A triangular fuzzy number $P = (a, b, c)$ is defined by its membership function is given below,

$$\mu_P(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{c-x}{c-b} & b \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.4. A triangular fuzzy number$P = (a, b, c) \in F(R)$ can also berepresented as a pair $P = (\underline{p}, \overline{p})$of functions which satisfies the following requirements:

$\underline{p}(n)$ is bounded monotonic increasing left continuous function.

$\overline{p}(n)$ is abounded monotonic decreasing left continuous function.

$$\underline{p}(n) \le \overline{p}(n), \quad 0 \le n \le 1$$

Definition 2.5: We denote this triangular fuzzy number by $P = (a, b, c) \in F(R)$Weuse $F(R)$to denote the set of all triangular fuzzy numbers Also if $m = b$ represents the modal value ormid point, $\gamma = (b - a)$represents the left spread and $\varepsilon = (c - a)$ represent the right spread of the triangular fuzzy number p□(a,b,c) can also be represented by p= (γ,m,ε).

Definition 2.6:

For an arbitrary triangular fuzzy number p$\in F(R)$ can also be represented as a pair p = $(\underline{p}, \overline{p})$ of functions $\underline{p}(n)$ and $\overline{p}\,(n)$ for $0 \le r \le 1$ which satisfies the following requirements

$\overline{p}(n)$ is a bounded monotonic increasing left continuous function.

$\underline{p}(n)$ is a bounded monotonic decreasing left continuous function.

$\overline{p}(n) \le \underline{p}(n)$,0 ≤ n ≤ 1

Definition 2.7:

For an arbitrary triangular fuzzy number p = $(\underline{p}, \overline{p})$ the number $m = \left(\frac{\underline{p}(n)+\overline{p}(n)}{2}\right)$ is said to a location index number of p The two non-decreasing left continuous functions γ=(m-$\underline{p}$), ε=($\overline{p} - m$)are called the left fuzziness index function and the right fuzziness index function respectively. Hence every triangular fuzzy number p=(a, b, c) can also be represented by p = (m,γ,ε)

Definition 2.5. Arithmetic operation on triangular Fuzzy Numbers Ming Ma et al. [11]have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the Fuzziness index functions are considered to follow the lattice rule, which is least upper bound in the lattice L. That is for p, q □L we define $p \vee q = \max\{p, q\}$and $p \wedge q = \min\{p, q\}$.

For arbitrary triangular fuzzy numbers $p = (a, b, c)$ and $q = (l, m, n)$and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular fuzzy numbers are defined by

Addition: $(a, b, c) + (l, m, n) = (a + l, \max\{b, m\}, \max\{c, n\})$

Subtraction:$(a, b, c) - (l, m, n) = (a - l, \min\{b, m\}, \min\{c, n\})$

Multiplication:$(a, b, c) * (l, m, n) = (a * l, \max\{b, m\}, \max\{c, n\})$

Division: $(a, b, c) \div (l, m, n) = (a \div l, \min\{b, m\}, \min\{c, n\})$

Methodology:

In this paper, we assume that the D Mhasal ready adopted triangular fuzzy numbers to represent the fuzzy market demand and supply chain costs. In practice, the DM are familiar with estimating optimistic, pessimistic and most likely parameters and the pattern of triangular distribution is commonly adopted due to ease in defining the maximum and minimum limit of deviation of the fuzzy number from its central value. The primary advantages of the triangular fuzzy number are the simplicity and flexibility of the fuzzy arithmetic operations (Liang, 2008).

In the process of defuzzification, there are many important measures to compare to fuzzy numbers, such as Hausdorff distance (Chaudhuri & Rosenfeld, 1999), Hamming distance (Diamond & Kloeden, 1994) and Jimenez (Jimenez, 2006). In this paper α−cut method will be used to approximate a fuzzy number.

Numerical Illustration:

To illustrate the solution approach, computational experiments are presented in this section. We consider a supply chain with wolves. In the higher level, we have manufacture eras a leader and in the lower level we have two distribution centers as two followers. Product 1 and product 2 are produced by the manufacturer, and then distributed them to two distribution centers. The numerical values of model parameters for manufacturer are given in Table.

Table 1: Data for the manufacturer

Parameters	value	Parameters	value	Parameters	value
a_1	$\tilde{50}$=(35,50,60)	c2	3	s_{11}	20
a_2	$\tilde{30}$=(24,30,50)	u_{11}	30	s21	10
b_{11}	$\tilde{15}$=(12,15,20)	u_{21}	30	s_{12}	20
b_{21}	$\tilde{10}$=(9,10,20)	u_{12}	20	s22	10
b_{12}	$\tilde{25}$=(20,25,30)	u22	20	R	5000
b_{22}	$\tilde{20}$=(16,20,25)	r_1	2	C	6000

Demand Value for Product 1 and 2

D1 is $\tilde{8}$20=(750,820,890) and D2 is $\tilde{4}$50=(400,450,550)

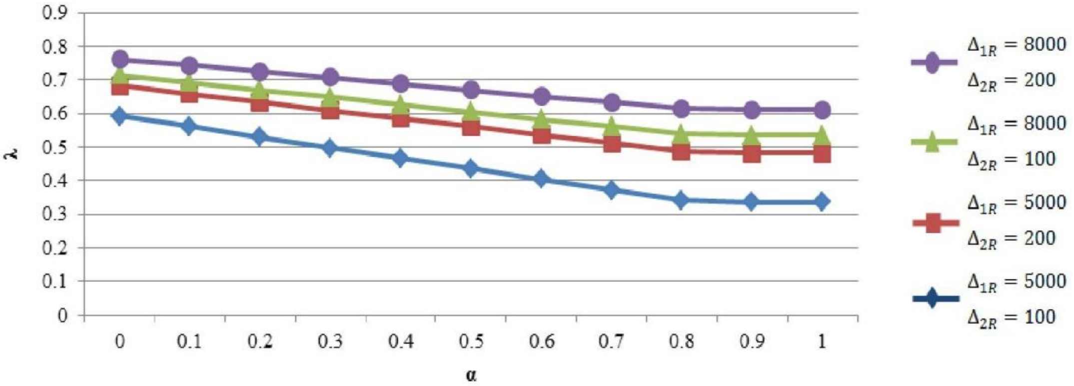
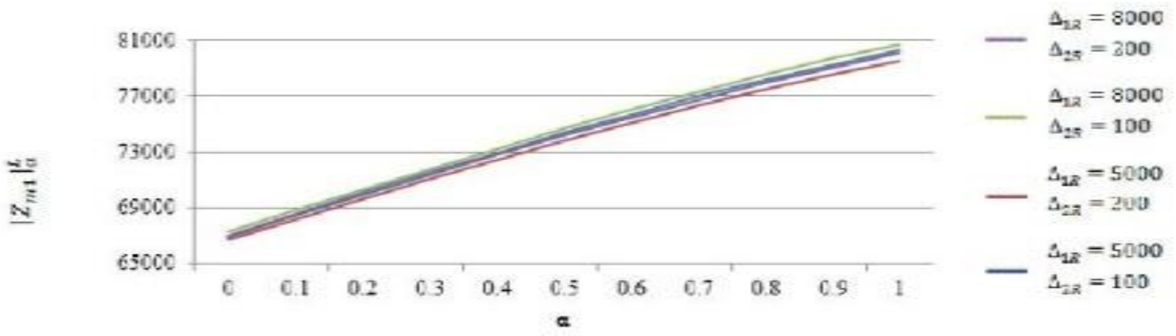
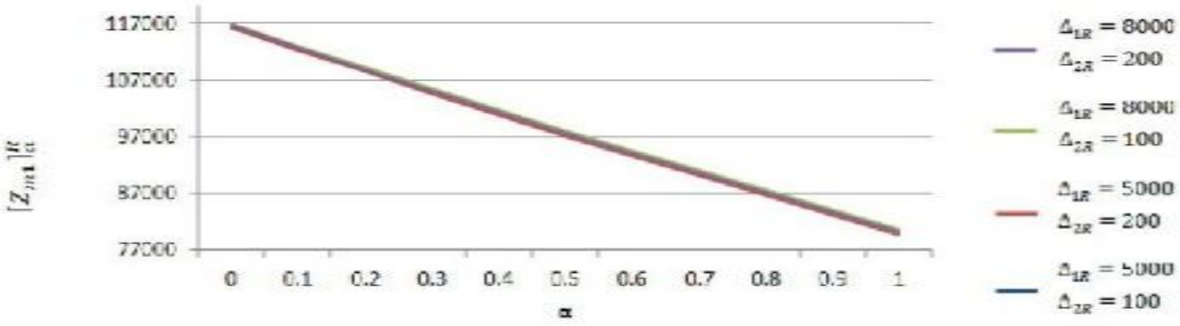


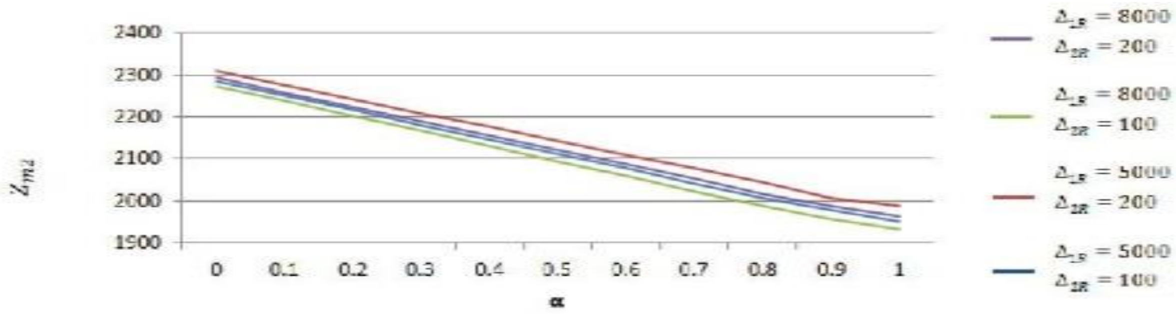
Fig. 3: Changes of λ vs. α



(4.a)



(4.b)



(4.c)

Conclusion:

This paper, supply chain optimization in a multi-objective multi-Multi decision linear bi-level problem with uncertain customer demand and costs was discussed. For this purpose, a supply chain with a manufacturer. Fuzzy objective functions and constraints are converted into crisp ones using fuzzy method. With using extended Kuhn–Tucker approach, bi-level problem is transformed into single level problem. Finally, we develop a fuzzy goal programming model to solve obtained multi- objective linear programming problem. A numerical example is presented to demonstrate the effectiveness of the model and solution method. According to predefined solution method, the example was solved and a set of Pareto-optimal solutions were obtained for choice of decision makers for choices that they maid.

Lastly, there are some possible directions for future research. In this paper, it is assumed that there isn't any shortage in any of supply chain levels. While we need to examine this issue in some real- world problems and inter costs associated with shortages and other constraints relevant to the problem. Also we have assumed that there is an uncooperative relationship between followers in the lower level where there is no sharing of decision variables among the followers. In such a situation, there are obviously neither shared objectives nor shared constraints among the followers. While different relationships among these followers excepting cooperative relationship like cooperative and partial cooperative could cause multiple different processes for deriving an optimal solution for the upper level's decision-making. So it can be considered according to model conditions.

Future scope:

To develop efficient algorithms and expand its applicability across diverse domains.

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