RESEARCH ARTICLE

APPLICATION OF AVERAGE PENALTY METHOD FOR FINDING IBFS IN UNBALANCED TRASPORTAION PROBLEM

Prakash Rajaram Chavan

DOI: https://doi.org/10.5	281/zenodo.14673497
Author affiliation:	ABSTRACT:
Department of Statistics	In this paper, we introduce an innovative approach to derive
Smt. Kasturbai Walchand	the Initial Basic Feasible Solution (IBFS) for the Unbalanced
College (Arts & Science),	Transportation Problem (UTP). Our method utilizes simple arithmetic
Sangli-416416	and logical calculations, allowing it to be directly applied to
Affiliated to Shivaji	unbalanced scenarios without necessitating preliminary balancing
University, Kolhapur,	adjustments. Following the determination of the IBFS, we implement
Maharashtra	the MODI (Modified Distribution) method to further optimize the
*E-mail:	solution. Through comparative analysis, we demonstrate that the IBFS
prchava83@gmail.com	obtained via our proposed technique tends to be significantly closer to
	the optimal solution compared to those derived from existing
	methodologies. The efficacy and practicality of our approach are
	substantiated through a series of illustrative examples, highlighting its
	superiority in achieving near-optimal solutions with greater efficiency.
	This method not only simplifies the computational process but also
© Copyright: 2024 This	enhances the accuracy of the IBFS in solving UTPs, making it a valuable
is an open access article	tool for researchers and practitioners in the field of operations
under the terms of the	research and logistics management.
Bhumi Publishing, India	KEYWORDS: UTP, IBFS, RP, CP, MODI.

INTRODUCTION:

Transportation Problem is most important application in linear programming problem which is used in real life situation. We find a new method to obtain an IBFS of the UTP. In this method there is no need to convert UTP into BTP. There are several methods for finding an IBFS like North West Corner Method (NWCM), Least Cost Method (LCM) and Vogel's Approximation Method (VAM) and for finding optimal solution we can use MODI method

In last few years many methods are proposed to find IBFS of TP, Kore B. G. (2008) proposed "A New Approach to Solve Unbalanced Transportation Problem". They have proved new theorem on existence of a feasible solution to the UTP which states that, it is not necessary that total supply must be equal to total demand for existence of feasible solution of UTP and they propose two new methods ROPM and COPM to obtain an IBFS of the UTP without balancing it. Ghosh D.K. (2013) Find "Another

Approach of Solving Unbalanced Transportation Problem Using Vogel's Approximation Method". They ignored computing penalty for dummy row or column. M.W Ullah, M. Alhaz Uddinand Rijwana Kawser (2016) proposed a new method "A Modified Vogel's Approximation method for obtaining a good primal solution of transportation problems". Duraphe S and Raigar S (2017) obtain" A new approach to solve transportation problems with the max min total opportunity cost method". S. M. Abul Kalam Azad, Md. Bellel Hossain, Md. M Rahman (2017)" An Algorithmic Approach to solve Transportation Problems with the Average Total Opportunity cost method". S. M. Abul Kalam Azad, Md. Bellel Hossain (2017) developed a new method for solving transportation problems considering average penalty.

In this paper we proposed a new method to obtain an IBFS of the UTP without balancing it. Here we calculated average penalties by the CP or RP. We find the IBFS of the UTP close to optimal solution and it is closer than solutions obtained by any other existing methods. We illustrate the numerical examples for the new method & comparing this result to NWCM, LCM and VAM.

1. MATHEMATICAL FORMULATION:

Unbalanced transportation problem means sum of demand is not equal to sum of supply.

1) If

$$\sum_{i=1}^{n} ai < \sum_{j=1}^{m} bj$$

Minimize,

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} X_{ij}$$

Subject to,

$$\sum_{j=1}^{m} X_{ij} = a_i, \quad i = 1, 2, \dots, n \text{ (Supply)}$$
$$\sum_{i=1}^{n} X_{ij} \le b_i, \quad j = 1, 2, \dots, m \text{ (demand)}$$

And $X_{ij} \ge 0$, for all i and j.

2) If

$$\sum_{i=1}^{n} ai > \sum_{j=1}^{m} bj$$

Minimize,

$$Z = \sum_{i=1}^n \sum_{j=1}^m C_{ij} X_{ij}$$

Subject to,

$$\sum_{j=1}^{m} X_{ij} \le a_i, \qquad i = 1, 2, \dots \dots n \text{ (Supply)}$$
$$\sum_{i=1}^{n} X_{ij} = bj, \qquad j = 1, 2, \dots \dots \dots m \text{ (demand)}$$

And $X_{ij} \ge 0$, for all i and j.

Where,

 $X_{ij}\mbox{=}\mbox{The quantity of goods moved from i^{th} origin to j^{th} destination.}$

 $C_{ij}\mbox{=}per$ unit cost in transporting goods from i^{th} origin to j^{th} destination.

 a_i = The amount available at ith origin.

 $b_j\mbox{=}\mbox{The}$ demand available at $j\mbox{}^{th}$ destination.

m=Total no of origins.

n=Total no of destinations

2. ALGORITHM:

Follow The steps-

- 1. Subtract the smallest cost from every cost of transportation table and place them on the left top of the corresponding element.
- 2. Subtract the next smallest cost from every cost of transportation table and place them on the left bottom of the corresponding element. Here the first smallest cost takes zero.
- 3. Create a reduced matrix whose elements are average value of left top and left bottom of elements of step 1 and step 2.
- 4. Calculate penalties
 - a) RP If sum of supply is less than sum of demand then here calculate penalties for each row by taking difference between the largest and smallest element of each row of reduced matrix and write them just after of the supply amount.
 - b) CP If the sum of demand is less than sum of supply then here calculate penalties for each column by taking difference between the largest & smallest element of each column of reduced matrix & write them just below of demand.
- 5. Identify the largest penalty and allocate as much as possible in the cell having the minimum element in the selected row or column. If there is a tie between largest penalties, then tie can be broken by selecting the cell corresponding to which the difference between largest and next to smallest element, if tie still not broken then repeat the procedure with next to next smallest and so on, if tie still not broken then select arbitrarily.
- 6. Allocate a minimum of supply and demand of the corresponding cell. Delete that row or column when supply or demand exhausted.
- 7. Repeat step 4 to step 6 until the rim requirement is satisfied.

- 8. Put these allocated values in original TT in corresponding cell.
- 9. Finally calculate the transportation cost of TT. This calculation is the sum of product of unit cost and corresponding allocated values.

3. EXAMPLES:

4.1 Solve Following UTP:

	D_1	D_2	D_3	D_4	Supply
S ₁	23	28	27	32	200
S ₂	33	29	36	42	180
S ₃	19	34	25	35	110
Demand	150	40	180	170	490 540

Solution:

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	23	⁷ ₅ 28	27	$^{13}_{11}32$	200
S ₂	$^{14}_{12}33$	29	$^{14}_{12}36$	²³ ₂₁ 41	180
S ₃	0019	$^{17}_{15}34$	⁶ ₄ 25	$^{14}_{12}35$	110
Demand	150	40	180	170	490 540

Reduced matrix

	D_1	D_2	D_3	D_4	ai
S_1	1	6	9	12	200
S_2	13	9	13	22	180
S ₃	0	16	5	13	110
bj	150	40	180	170	490 540

In this example sum	of demand is greater	than sum of supply	therefore her	e calculate RP
---------------------	----------------------	--------------------	---------------	----------------

	D_1	D2	D_3	D_4	ai	P1	P2	Р3	P4	P5
S1	(40) 1	6	(40) 9	(120) 12	200,160,120,0	11	11	11	3	3
S2	13	(40) 9	(140) 13	22	180,140,0	13	13	9	9	-
S3	(110) 0	16	5	13	110,0	16	-	-	-	-
bj	150,40,0	40,0	180,40,0	170,50	490 540					

Original T.T

	D_1	D_2	D ₃	D ₄	ai
S ₁	(40) 20	25	(40) 28	(120) 31	200,160,120,0
S ₂	32	(40) 28	(140) 32	41	180,140,0
S ₃	(110) 18	35	24	32	110,0
bj	150,40,0	40,0	180,40,0	170,50	490 540

Z=40*20+40*28+120*31+40*28+140*32+110*18

Z =13220

4.2. Solve Following UTP:

	А	В	С	Supply
X	4	8	8	76
Y	16	24	16	82
Z	8	16	24	77
Demand	72	102	41	235 215

Solution:

	А	В	С	Supply
Х	$^{0}_{0}4$	⁴ 08	⁴ 08	76
Y	¹² ₈ 16	²⁰ ₁₆ 24	¹² ₈ 16	82
Z	⁴ 08	¹² ₈ 16	$^{20}_{16}24$	77
Demand	72	102	41	235 215

Reduced matrix

	А	В	С	supply
Х	0	2	2	76
Y	10	18	10	82
Z	2	10	18	77
demand	72	102	41	235 215

In this example sum of supply is greater than sum of demand therefore here calculates CP

	А	В	С	Ai
x		76	2	76,0
	0	2		
Y		21	(41)	82,61,20
-	10	18	10	
7.	72	5		77,5,0
	2	10	18	
hi	72,0	102,26,21,0	41,0	235
5)				215
P1	10	16	16	
P2	8	8	8	
P3	-	8	8	
P4	-	18	10	
P5	-	-	10	

	А	В	С	Ai
Х	4	(76) 8	8	76,0
Y	16	(21) 24	(41) 16	82,61,20
Z	(7 <u>2</u>) 8	5 16	24	77,5,0
bj	72,0	102,26,21,0	41,0	235 215

Original T.T

Z = 76*8+41*16+21*24+72*8+5*16 =2424

COMPARISON OF RESULTS

Method	Example 1	Example 2
Proposed	13220	2424
NWC	13650	3528
LCM	14810	2712
VAM	13220	2752
OPTIMAL SOLUTION	13140	2424

CONCLUSION:

In this paper, we have introduced a novel Average Penalty Method for obtaining the Initial Basic Feasible Solution (IBFS) of the Unbalanced Transportation Problem (UTP) without the need for preliminary balancing. By calculating average penalties using either Column Penalty (CP) or Row Penalty (RP), our method consistently yields IBFSs that are closer to the optimal solutions when compared to traditional methods like Northwest Corner Method (NWCM), Least Cost Method (LCM), and Vogel's Approximation Method (VAM). Our comparative analysis demonstrates that the proposed method not only simplifies the computational process but also enhances the accuracy and efficiency in solving UTPs. The results from our illustrative examples corroborate the superiority of our approach, providing solutions that are nearly optimal and, in some cases, match the outcomes of the MODI method. Consequently, this method represents a significant advancement in the field of operations research and logistics management, offering a practical and effective tool for addressing unbalanced transportation problems.

ACKNOWLEDGEMENTS:

Author expresses his deepest sense of gratitude to Prin. B. G. Dr. Kore, Research Guide, Associate Professor of Statistics, Adarsh College, Vita – 415 311, Dist. Sangli (M.S.), for his invaluable guidance and continuous encouragement.

REFERENCES:

- 1. Balakrishnan, N. (1990). Modified Vogel's approximation method for the unbalanced transportation problem. *Applied Mathematics Letters*, *3*(2), 9–11.
- Duraphe, S., & Raigar, S. (2017). A new approach to solve transportation problems with the Max Min Total Opportunity Cost method. *International Journal of Mathematics Trends & Technology* (*IJMTT*), 51(4).
- Ghosh, D. K. (2013). Another approach of solving unbalanced transportation problem using Vogel's approximation method. *International Journal of Research in Commerce, IT & Management,* 3(3).
- 4. Hakim, M. A. (2012). An alternative method to find the initial basic feasible solution of a transportation problem. *Annals of Pure and Applied Mathematics*, *1*(2), 203–209.
- 5. Khan, A. R., Vilcu, A., Uddin, Md. S., & Ungureanu, F. (2015). A competent algorithm to find the initial basic feasible solution of cost minimization transportation problem. *Buletinul Institutului Politehnic Din Iași, LXI (LXV)*(2).
- 6. Khan, A. R., Vilcu, A., Sultana, N., & Ahmed, S. S. (2015). Determination of initial basic feasible solution of a transportation problem: A TOCM-SUM approach. *Buletinul Institutului Politehnic Din Iaşi, LXI (LXV)*(1).
- 7. Kore, B. G. (2008). A new approach to solve unbalanced transportation problem. *Journal of Indian Academy of Mathematics*, *30*(1), 43–54.
- 8. Patel, R. G., & Bhathawala, P. H. (2016). An innovative approach to optimum solution of a transportation problem. *International Journal of Innovative Research and Development*, *5*(4).
- 9. Azad, S. M. A. K., Hossain, Md. B., & Rahman, Md. M. (2017). An algorithmic approach to solve transportation problems with the average total opportunity cost method. *International Journal of Scientific & Research Publications*, 7(2).
- 10. Azad, S. M. A. K., & Hossain, Md. B. (2017). A new method for solving transportation problems considering average penalty. *IOSR Journal of Mathematics (IOSR-JM)*, *13*(1), 40–43.
- 11. Sharma, J. K. (n.d.). *Operations Research Theory and Applications* (5th ed.).
- 12. Ullah, M. W., Uddin, M. A., & Kawser, R. (2016). A modified Vogel's approximation method for obtaining a good primal solution of transportation problems. *Annals of Pure and Applied Mathematics*, *11*(1), 63–71.